Frequent Itemset Mining & Association Rules

CS246: Mining Massive Datasets
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Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers
- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items
- **A classic rule:**
  - If one buys diaper and milk, then he is likely to buy beer
  - Don't be surprised if you find six-packs next to diapers!

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

The Market-Basket Model

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
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5	Coke, Diaper, Milk

- A large set of ***items***
  - e.g., things sold in a supermarket
- A large set of ***baskets***, each is a small subset of items
  - e.g., the things one customer buys on one day
- **A general many-many mapping (association) between two kinds of things**
  - But we ask about connections among "items", not "baskets"

Association Rules: Approach

- **Given a set of baskets**
- Want to discover **association rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - Amazon!
- **2 step approach:**
  - **1)** Find frequent **itemsets**
  - **2)** Generate **association rules**

Input:

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

Applications – (1)

- **Items** = products; **Baskets** = sets of products someone bought in one trip to the store
- **Real market baskets**: Chain stores keep TBs of data about what customers buy together
  - Tells how typical customers navigate stores, lets them position tempting items
  - Suggests tie-in "tricks", e.g., run sale on diapers and raise the price of beer
  - High **support** needed, or no $$'s
- **Amazon's people who bought *X* also bought *Y***

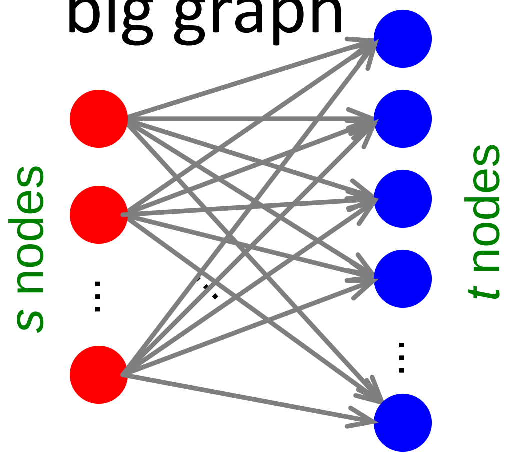
Applications – (2)

- **Baskets** = sentences; **Items** = documents containing those sentences
  - Items that appear together too often could represent plagiarism
  - Notice items do not have to be "in" baskets

- **Baskets** = patients; **Items** = drugs & side-effects
  - Has been used to detect combinations of drugs that result in particular side-effects
  - **But requires extension:** Absence of an item needs to be observed as well as presence

Applications – (3)

- **Finding communities in graphs (e.g., web)**
- **Baskets** = nodes; **Items** = outgoing neighbors
  - Searching for complete bipartite subgraphs $K_{s,t}$ of a big graph



A dense 2-layer graph

Use this to define topics:
What the same people on the left talk about on the right

- **How?**
  - View each node i as a basket B_i of nodes i it points to
  - $K_{s,t}$ = a set Y of size t that occurs in s buckets B_i
  - Looking for $K_{s,t}$ → set of support s and look at layer t – all frequent sets of size t

Outline

First: Define

- Frequent itemsets
- Association rules:
  - Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

- Finding frequent pairs
- Apriori algorithm
- PCY algorithm + 2 refinements

Frequent Itemsets

- **Simplest question:** Find sets of items that appear together "frequently" in baskets
- ***Support*** for itemset I: Number of baskets containing all items in I
  - Often expressed as a fraction of the total number of baskets
- Given a ***support threshold*** s, then sets of items that appear in at least s baskets are called ***frequent itemsets***

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Support of {Beer, Bread} = 2

Example: Frequent Itemsets

- **Items** = {milk, coke, pepsi, beer, juice}
- **Minimum support** = 3 baskets

B_1 = {m, c, b} B_2 = {m, p, j}
B_3 = {m, b} B_4= {c, j}
B_5 = {m, p, b} B_6 = {m, c, b, j}
B_7 = {c, b, j} B_8 = {b, c}

- **Frequent itemsets:** {m}, {c}, {b}, {j},
  {m,b} , {b,c} , {c,j}.

Association Rules

- **Association Rules:**
  If-then rules about the contents of baskets
- $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is ***likely*** to contain j"
- **In practice there are many rules, want to find significant/interesting ones!**
- ***Confidence*** of this association rule is the probability of j given $I = \{i_1, \ldots, i_k\}$

$$\mathrm{conf}(I \rightarrow j) = \frac{\mathrm{support}(I \cup j)}{\mathrm{support}(I)}$$

Interesting Association Rules

- **Not all high-confidence rules are interesting**
  - The rule $X \to milk$ may have high confidence for many itemsets X, because milk is just purchased very often (independent of X) and the confidence will be high
- **Interest** of an association rule $I \to j$: difference between its confidence and the fraction of baskets that contain j

$$\text{Interest}(I \to j) = \text{conf}(I \to j) - \Pr[j]$$

  - Interesting rules are those with high positive or negative interest values
  - For uninteresting rules the fraction of baskets containing *j* will be the same as the fraction of the subset baskets including {*I*, *j*}. So, confidence will be high, interest low.

Example: Confidence and Interest

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, b\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- **Association rule: {m, b} $\rightarrow$ c**
  - **Confidence** = 2/4 = 0.5
  - **Interest** = $|0.5 – 5/8|$ = 1/8
    - Item *c* appears in 5/8 of the baskets
    - Rule is not very interesting!

Finding Association Rules

- **Problem: Find all association rules with support $\geq s$ and confidence $\geq c$**
  - **Note:** Support of an association rule is the support of the set of items on the left side
- **Hard part: Finding the frequent itemsets!**
  - If $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, \ldots, i_k\}$ and $\{i_1, i_2, \ldots, i_k, j\}$ will be "frequent"

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Mining Association Rules

- **Step 1:** Find all frequent itemsets I
  - (we will explain this next)
- **Step 2: Rule generation**
  - For every subset A of I, generate a rule $A \to I \setminus A$
    - Since I is frequent, A is also frequent
    - **Variant 1:** Single pass to compute the rule confidence
      - conf(A,B→C,D) = supp(A,B,C,D)/supp(A,B)
    - **Variant 2:**
      - **Observation:** If A,B,C→D is below confidence, so is A,B→C,D
      - Can generate "bigger" rules from smaller ones!
  - **Output the rules above the confidence threshold**

Example

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, c, b, n\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- Min support s=3, confidence c=0.75
- **1) Frequent itemsets:**
  - {b,m} {b,c} {c,m} {c,j} {m,c,b}
- **2) Generate rules:**
  - ~~b→m: c=4/6~~ b→c: c=5/6 ~~b,c→m: c=3/5~~
  - m→b: c=4/5 … b,m→c: c=3/4
  - ~~b→c,m: c=3/6~~

Compacting the Output

1. ***Maximal Frequent itemsets***:
   no immediate superset is frequent

2. ***Closed itemsets***:
   no immediate superset has the same count (> 0).
   - Stores not only frequent information, but exact counts

Example: Maximal/Closed

	Count	Maximal (s=3)	Closed
A	4	No	No
B	5	No	Yes
C	3	No	No
AB	4	Yes	Yes
AC	2	No	No
BC	3	Yes	Yes
ABC	2	No	Yes

Frequent, but superset BC also frequent.

Frequent, and its only superset, ABC, not freq.

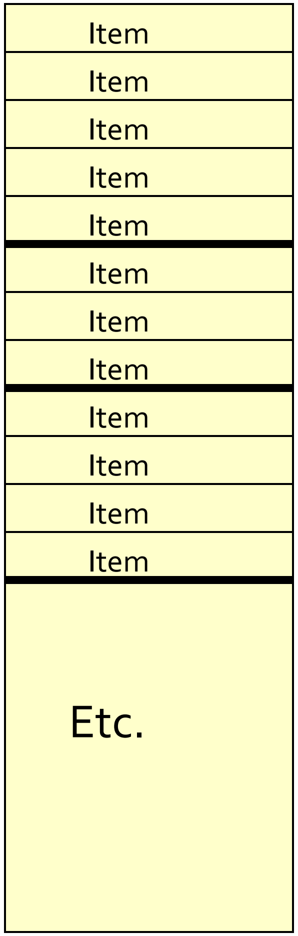
Superset BC has same count.

Its only super-set, ABC, has smaller count.

Finding Frequent Itemsets

Itemsets: Computation Model

- Back to finding frequent itemsets
- Typically, data is kept in flat files rather than in a database system:
  - Stored on disk
  - Stored basket-by-basket
  - Baskets are **small** but we have many baskets and many items
    - Expand baskets into pairs, triples, etc. as you read baskets
    - Use ***k*** nested loops to generate all sets of size ***k***



Items are positive integers, and boundaries between baskets are –1.

Note: We want to find frequent itemsets. To find them, we have to count them. To count them, we have to generate them.

Computation Model

- The true cost of mining disk-resident data is usually the number of disk I/O's
- In practice, association-rule algorithms read the data in *passes* – all baskets read in turn
- We measure the cost by the **number of passes** an algorithm makes over the data

Main-Memory Bottleneck

- For many frequent-itemset algorithms, **main-memory** is the critical resource
  - As we read baskets, we need to count something, e.g., occurrences of pairs of items
  - The number of different things we can count is limited by main memory
  - Swapping counts in/out is a disaster (why?)

Finding Frequent Pairs

- **The hardest problem often turns out to be finding the frequent pairs of items $\{i_1, i_2\}$**
  - Why? Often frequent pairs are common, frequent triples are rare
    - **Why?** Probability of being frequent drops exponentially with size; number of sets grows more slowly with size.
- Let's first concentrate on pairs, then extend to larger sets
- **The approach:**
  - We always need to generate all the itemsets
  - But we would only like to count/keep track of those itemsets that in the end turn out to be frequent

Naïve Algorithm

- **Naïve approach to finding frequent pairs**
- Read file once, counting in main memory the occurrences of each pair:
  - From each basket of *n* items, generate its *n(n-1)/2* pairs by two nested loops
- **Fails if $(\#\text{items})^2$ exceeds main memory**
  - **Remember:** #items can be 100K (Wal-Mart) or 10B (Web pages)
    - Suppose 10^5 items, counts are 4-byte integers
    - Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$
    - Therefore, $2*10^{10}$ (20 gigabytes) of memory needed

Counting Pairs in Memory

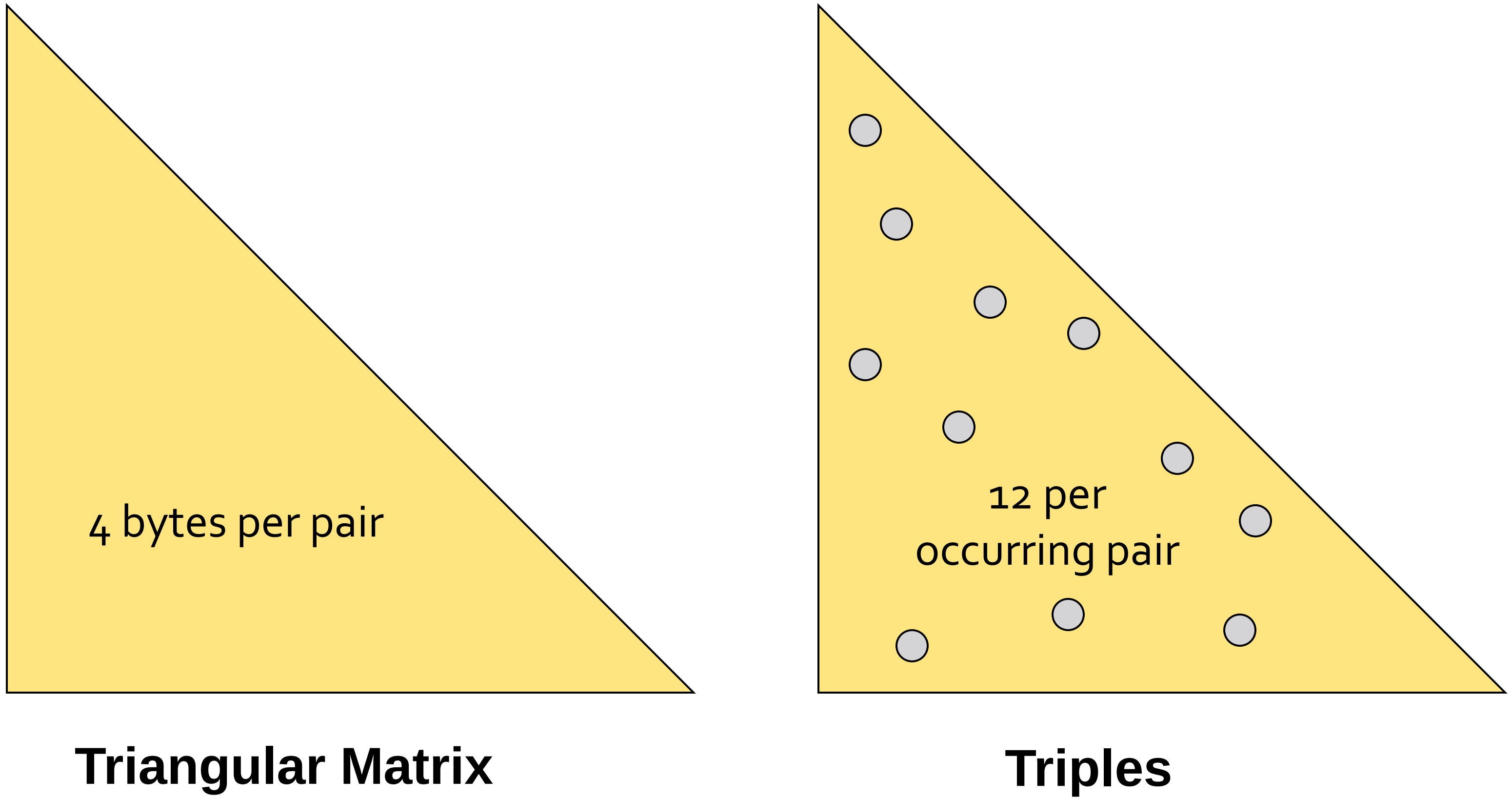
Two Approaches:

- **Approach 1:** Count all pairs using a matrix
- **Approach 2:** Keep a table of triples [i, j, c] = "the count of the pair of items {i, j} is c."
  - If integers and item ids are 4 bytes, we need approximately 12 bytes for pairs with count > 0
  - Plus some additional overhead for the hashtable

Note:

- **Approach 1** only requires 4 bytes per pair
- **Approach 2** uses 12 bytes per pair (but only for pairs with count > 0)

Comparing the 2 Approaches

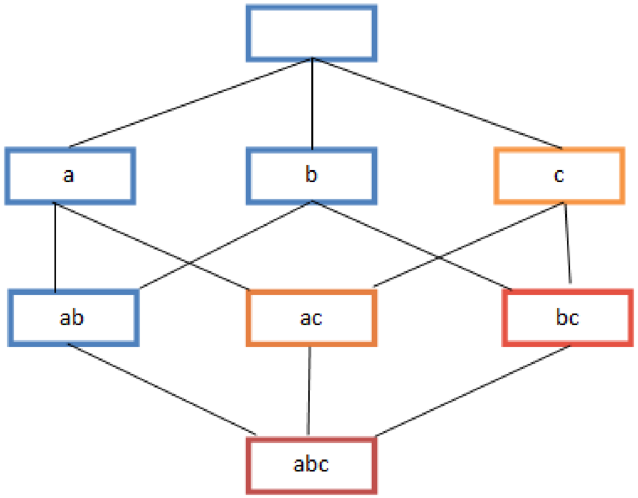


Triangular Matrix Approach

- **Triangular Matrix Approach**
  - n = total number items
  - Count pair of items $\{i, j\}$ only if $i<j$
- Keep pair counts in lexicographic order:
  - $\{1,2\}, \{1,3\},\ldots, \{1,n\}, \{2,3\}, \{2,4\},\ldots,\{2,n\}, \{3,4\},\ldots$
- Pair $\{i, j\}$ is at position $(i-1)(n-i/2) + j-1$
- Total number of pairs $n(n-1)/2$; total bytes= $2n^2$
- **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses 12 *bytes* per pair *(but only for pairs with count > 0)*
  - Beats triangular matrix if less than 1/3 of possible pairs actually occur

A-Priori Algorithm

A-Priori Algorithm – (1)

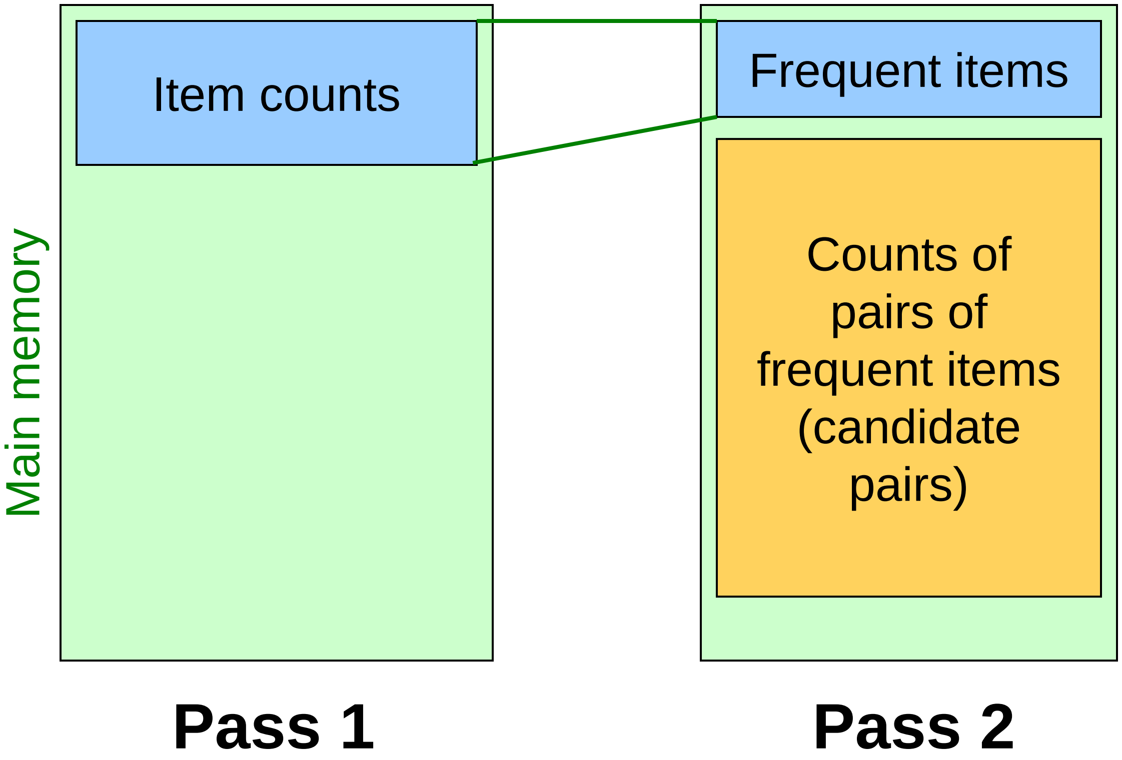


- A **two-pass** approach called ***a-priori*** limits the need for main memory
- **Key idea: *monotonicity***
  - If a set of items I appears at least s times, so does every **subset** J of I.
- **Contrapositive for pairs:**
  If item i does not appear in s baskets, then no pair including i can appear in s baskets

A-Priori Algorithm – (2)

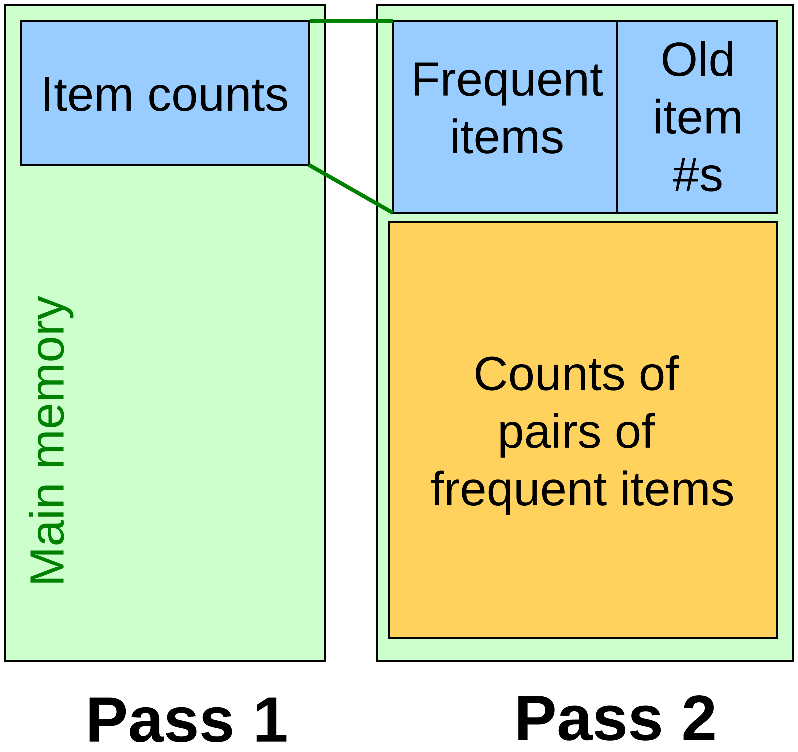
- **Pass 1:** Read baskets and count in main memory the occurrences of each individual item
  - Requires only memory proportional to #items
- **Items that appear at least *s* times are the frequent items**
- **Pass 2:** Read baskets again and count in main memory only those pairs where both elements are frequent (from Pass 1)
  - Requires memory proportional to square of **frequent** items only (for counts)
  - Plus a list of the frequent items (so you know what must be counted)

Main-Memory: Picture of A-Priori



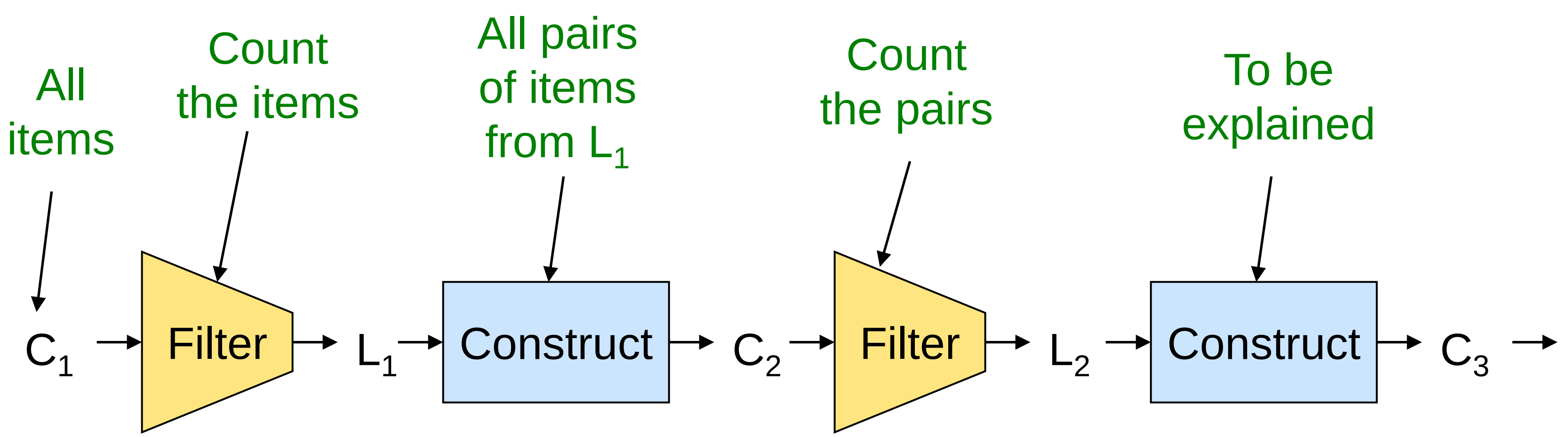
Detail for A-Priori

- You can use the triangular matrix method with ***n*** = number of frequent items
  - May save space compared with storing triples
- **Trick:** re-number frequent items 1,2,… and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- **For each *k*, we construct two sets of *k-tuples*** (sets of size *k*):
  - C_k = ***candidate k-tuples*** = those that might be frequent sets (support $\geq$ s) based on information from the pass for k–1
  - L_k = the set of truly frequent k-tuples



Example

Note that one can be more careful here with rule generation. For example, we know {b,m,j} cannot be frequent since {m,j} is not frequent

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }
  - Count the support of itemsets in C_1
  - Prune non-frequent: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent: L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} {b,c,j} {b,m,j} {c,m,j} }
  - Count the support of itemsets in C_3
  - Prune non-frequent: L_3 = { {b,c,m} }

A-Priori for All Frequent Itemsets

- One pass for each *k* (itemset size)
- Needs room in main memory to count each candidate *k*–tuple
- For typical market-basket data and reasonable support (e.g., 1%), *k* = 2 requires the most memory

- **Many possible extensions:**
  - Lower the support *s* as itemset gets bigger
  - Association rules with intervals:
    - For example: Men over 65 have 2 cars
  - Association rules when items are in a taxonomy
    - Bread, Butter $\rightarrow$ FruitJam
    - BakedGoods, MilkProduct $\rightarrow$ PreservedGoods

PCY (Park-Chen-Yu) Algorithm

PCY (Park-Chen-Yu) Algorithm

- **Observation:**
  In pass 1 of a-priori, most memory is idle
  - We store only individual item counts
  - **Can we use the idle memory to reduce memory required in pass 2?**
- **Pass 1 of PCY:** In addition to item counts, maintain a hash table with as many buckets as fit in memory
  - Keep a count for each bucket into which **pairs** of items are hashed
    - Just the count, not the pairs that hash to the bucket!

PCY Algorithm – First Pass

```
FOR (each basket) :
    FOR (each item in the basket) :
        add 1 to item's count;
    FOR (each pair of items) :
        hash the pair to a bucket;
        add 1 to the count for that bucket;
```

New in PCY

- Pairs of items need to be generated from the input file; they are not present in the file
- We are not just interested in the presence of a pair, but we need to see whether it is present at least *s* (support) times

Observations about Buckets

- **If a bucket contains a frequent pair, then the bucket is surely frequent**
  - But we cannot use the hash to eliminate any member of this bucket
- Even without any frequent pair, a bucket can still be frequent
- **But, for a bucket with total count less than s, none of its pairs can be frequent**
  - Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- **Pass 2:**
  Only count pairs that hash to frequent buckets

PCY Algorithm – Between Passes

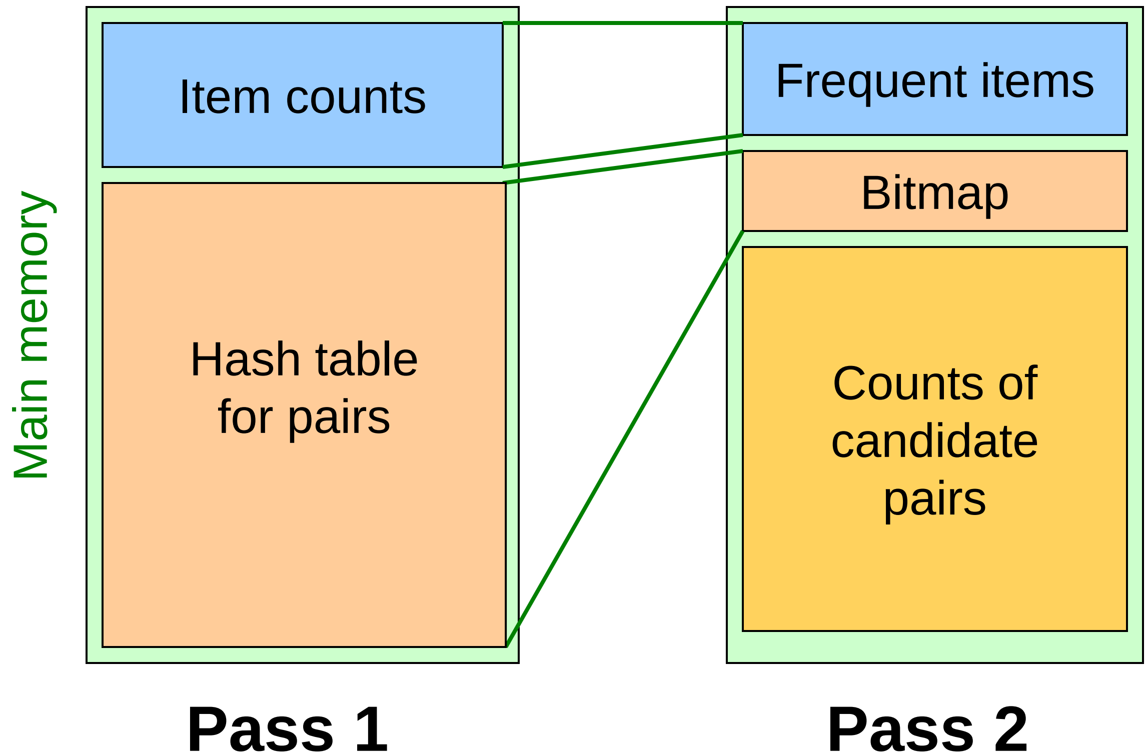
- **Replace the buckets by a bit-vector:**
  - 1 means the bucket count exceeded the support s (a *frequent bucket*); 0 means it did not
- **4-byte integer counts are replaced by bits, so the bit-vector requires 1/32 of memory**
- Also, decide which items are frequent and list them for the second pass

PCY Algorithm – Pass 2

- Count all pairs $\{i, j\}$ that meet the conditions for being a **candidate pair**:
    1. Both i and j are frequent items
    2. The pair $\{i, j\}$ hashes to a bucket whose bit in the bit vector is 1 (i.e., frequent bucket)

- **Both conditions are necessary for the pair to have a chance of being frequent**

Main-Memory: Picture of PCY



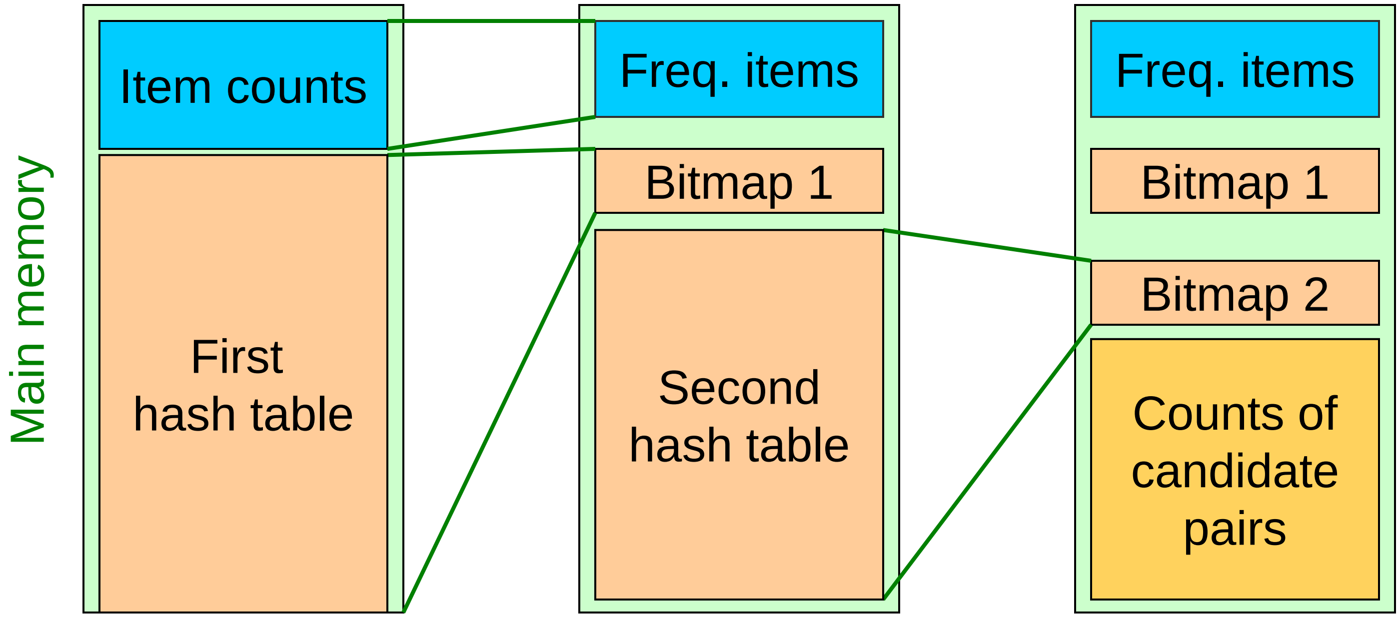
Main-Memory Details

- **Buckets require a few bytes each:**
  - **Note:** we don't have to count past s
  - #buckets is O(main-memory size)
- On second pass, a table of (item, item, count) triples is essential (we cannot use triangular matrix approach, why?)
  - Thus, hash table must eliminate approx. 2/3 of the candidate pairs for PCY to beat a-priori.

Refinement: Multistage Algorithm

- **Limit the number of candidates to be counted**
  - **Remember:** Memory is the bottleneck
  - Still need to generate all the itemsets but we only want to count/keep track of the ones that are frequent
- **Key idea:** After Pass 1 of PCY, rehash only those pairs that **qualify** for Pass 2 of PCY
  - *i* and *j* are frequent, and
  - *{i, j}* hashes to a frequent bucket from Pass 1
- On middle pass, fewer pairs contribute to buckets, so fewer ***false positives***
- **Requires 3 passes over the data**

Main-Memory: Multistage



Pass 1

Count items
Hash pairs {i,j}

Pass 2

Hash pairs {i,j}
into Hash2 iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1

Pass 3

Count pairs {i,j} iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1
{i,j} hashes to
freq. bucket in B2

Multistage – Pass 3

- **Count only those pairs $\{i, j\}$ that satisfy these candidate pair conditions:**
  1. Both i and j are frequent items
  2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is 1.
  3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is 1.

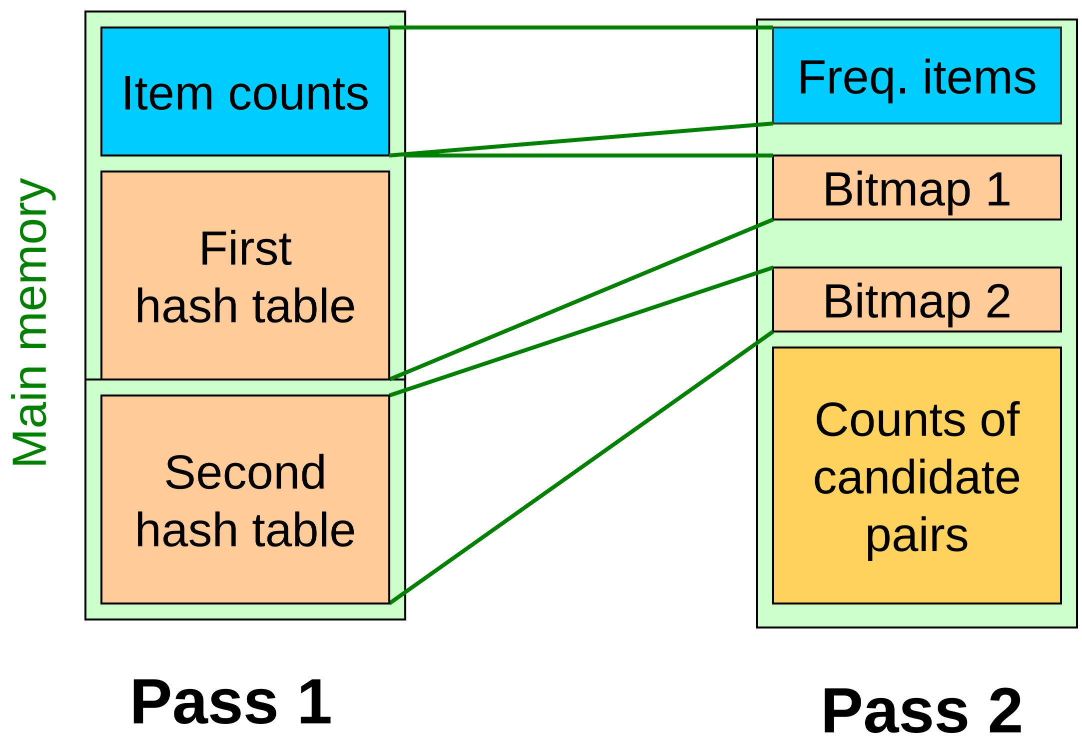
Important Points

1. **The two hash functions have to be independent**
2. **We need to check both hashes on the third pass**
   - If not, we would end up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket

Refinement: Multihash

- **Key idea:** Use several independent hash tables on the first pass
- **Risk:** Halving the number of buckets doubles the average count
  - We have to be sure most buckets will still not reach count s
- If so, we can get a benefit like multistage, but in only 2 passes

Main-Memory: Multihash



PCY: Extensions

- Either **multistage** or **multihash** can use more than two hash functions

- In **multistage**, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory

- For **multihash**, the bit-vectors occupy exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$

Frequent Itemsets in $\leq$ 2 Passes

Frequent Itemsets in ≤ 2 Passes

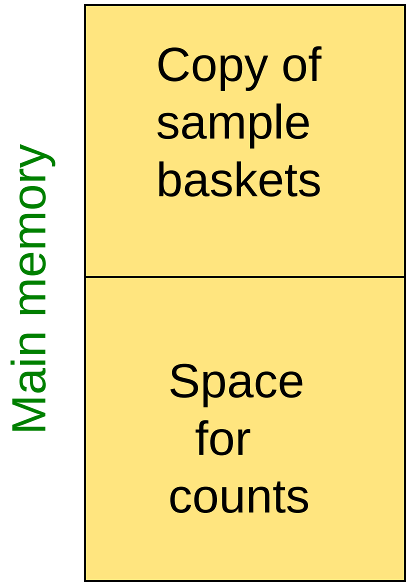
- A-Priori, PCY, etc., take *k* passes to find frequent itemsets of size *k*

- **Can we use fewer passes?**

- Use 2 or fewer passes for all sizes, but may miss some frequent itemsets
  - Random sampling
  - SON (Savasere, Omiecinski, and Navathe)
  - Toivonen (see textbook)

Random Sampling (1)

- Take a random sample of the market baskets
- Run a-priori or one of its improvements in main memory
  - So we don't pay for disk I/O each time we increase the size of itemsets
  - Reduce support threshold proportionally to match the sample size



Random Sampling (2)

- Optionally, verify that the candidate pairs are truly frequent in the entire data set by a second pass (avoid false positives)

- But you don't catch sets frequent in the whole but not in the sample
  - Smaller threshold, e.g., $s/125$, helps catch more truly frequent itemsets
    - But requires more space

SON Algorithm – (1)

- Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets
  - Note: we are not sampling, but processing the entire file in memory-sized chunks
- An itemset becomes a candidate if it is found to be frequent in *any* one or more subsets of the baskets.

SON Algorithm – (2)

- On a **second pass**, count all the candidate itemsets and determine which are frequent in the entire set
- **Key "monotonicity" idea:** an itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.

SON – Distributed Version

- SON lends itself to distributed data mining

- Baskets distributed among many nodes
  - Compute frequent itemsets at each node
  - Distribute candidates to all nodes
  - Accumulate the counts of all candidates

SON: Map/Reduce

- **Phase 1:** Find candidate itemsets
  - Map?
  - Reduce?

- **Phase 2:** Find true frequent itemsets
  - Map?
  - Reduce?