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Seek distances in disks with two independent heads per surface

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Abstract

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Estimates for the average seek distance in disk systems with two independent heads are derived. Comparison is made with conventional one-headed systems, as well as with disk systems having two heads at a fixed distance.

Keywords: Operating systems, performance evaluation

1. Introduction

Two-headed disk systems are commercially available, however their special characteristics are not broadly studied. As categorized in [5], these systems may have (a) two arms autonomously controlled, (b) two arms and a single controller or, finally, (c) a single arm with two fixed heads. The last case is examined in [1,3,6–8], the second case is examined in [2], while the first case is examined, to the authors' knowledge, only in [4]. In this paper a disk system with two arms autonomously controlled is considered. More specifically, in Section 2 it is assumed that, when a random request arrives, the two read/write heads are randomly positioned on the disk surface, while in Section 3 it is assumed that the heads do not take random positions but some scheduling is implemented. These sections derive formulas for the mean seek distance traveled to answer a random request. Finally, comparison is made with conventional single-headed disk systems and disk systems with two heads separated by a fixed number of tracks.

2. Analysis for randomly positioned heads

Consider that a disk is a linear storage medium consisting of C cylinders. On top of this linear medium two heads, named A and B, are moving back and forth to service the user or system requests. Both heads may move from the first to the last cylinder. Suppose, now, that at some point in time an arriving request hits cylinder X. The head with the minimum distance from the hit cylinder is chosen to move, while the other waits for the next demand.

Assume that both heads may lie on top of any cylinder with equal probability and that all cylinders are hit under a uniform probability distribution (this assumption will be discussed later in the last section). It is easy to see that the mean seek distance traveled is equal to the ratio of the sum of seek distances divided by the number of all the possible combinations of head positions and hit cylinders. First, we are going to calculate the numerator. That is:

$$\sum_{A=1}^{C}\sum_{B=1}^{C}\sum_{X=1}^{C}\min(|X-A|, |X-B|) \tag{1}$$

$$= \sum_{A=1}^{C}\sum_{B=1}^{A-1}\sum_{X=1}^{B-1}\min(|X-A|, |X-B|) + \sum_{A=1}^{C}\sum_{B=1}^{A-1}\sum_{X=B+1}^{C}\min(|X-A|, |X-B|)$$
$$+ \sum_{A=1}^{C}\sum_{B=A}^{C}\sum_{X=1}^{B-1}\min(|X-A|, |X-B|) + \sum_{A=1}^{C}\sum_{B=A}^{C}\sum_{X=B+1}^{C}\min(|X-A|, |X-B|). \tag{2}$$

We calculate each of the four triple summations of expression (2) separately. The first one yields:

$$\sum_{A=1}^{C}\sum_{B=1}^{A-1}\sum_{X=1}^{B-1}\min(|X-A|, |X-B|) = \sum_{A=1}^{C}\sum_{B=1}^{A-1}\sum_{X=1}^{B-1}(B-X) = \sum_{A=1}^{C}\sum_{B=1}^{A-1}(B^2-B)/2$$
$$= \sum_{A=1}^{C}(A^3-3A^2+2A)/6$$
$$= (C-2)(C-1)C(C+1)/24. \tag{3}$$

For the second triple summation we have:

$$\sum_{A=1}^{C}\sum_{B=1}^{A-1}\sum_{X=B+1}^{C}\min(|X-A|, |X-B|)$$
$$= \sum_{A=1}^{C}\sum_{B=1}^{A-1}\left(\sum_{X=B+1}^{MID}(X-B) + \sum_{X=MID+1}^{A-1}(A-X) + \sum_{X=A+1}^{C}(X-A)\right) \tag{4}$$

where $MID = \lfloor (A+B)/2 \rfloor$. The expression in the parenthesis equals:

$$(MID-B)(MID-B+1)/2$$
$$+(A-MID-1)(A-MID)/2 + (C-A)(C-A+1)/2. \tag{5}$$

If $A+B$ is even, then

$$2MID = A+B. \tag{6}$$

By substituting expressions (5, 6) in expression (4) we get:

$$\sum_{A=1}^{C}\sum_{B=1}^{A-1}\left(B^2-2AB+A^2+2(C-A)(C-A+1)\right)/4$$
$$= \sum_{A=1}^{C}\left(14A^3-A^2(24C+27)+A(12C^2+36C+13)-12C(C+1)\right)/24$$
$$= (3C-4)(C-1)C(C+1)/48. \tag{7}$$

If $A+B$ is odd, then

$$2MID = A + B - 1. \tag{8}$$

By substituting expressions (5,8) in expression (4) we get:

$$\sum_{A=1}^{C} \sum_{B=1}^{A-1} (B^2 - 2AB + A^2 + 2(C-A)(C-A+1) - 1)/4$$
$$= \sum_{A=1}^{C} (14A^3 - A^2(24C+27) + A(12C^2+36C+7) - 6(2C^2+2C-1))/24$$
$$= (C-2)(C-1)C(3C+5)/48. \tag{9}$$

For the third triple summation we have:

$$\sum_{A=1}^{C} \sum_{B=A}^{C} \sum_{X=1}^{B-1} \min(|X-A|, |X-B|)$$
$$= \sum_{A=1}^{C} \sum_{B=A}^{C} \left(\sum_{X=1}^{A-1} (A-X) + \sum_{X=A+1}^{MID} (X-A) + \sum_{X=MID+1}^{B-1} (B-X) \right). \tag{10}$$

The expression in the parenthesis is equal to:

$$A(A-1)/2 + (MID-A)(MID-A+1)/2 + (B-MID)(B-MID-1)/2. \tag{11}$$

If $A+B$ is even, then by substituting expressions (6, 11) in expression (10) we get:

$$\sum_{A=1}^{C} \sum_{B=A}^{C} (B^2 - 2AB + 3A^2 - 2A)/4$$
$$= \sum_{A=1}^{C} (-14A^3 + 9A^2(2C+3) - A(6C^2+18C+13) + C(2C^2+3C+1))/24$$
$$= (3C+4)(C-1)C(C+1)/48. \tag{12}$$

If $A+B$ is odd, then by substituting expressions (8, 11) in expression (10) we get:

$$\sum_{A=1}^{C} \sum_{B=A}^{C} (B^2 - 2AB + 3A^2 - 2A - 1)/4$$
$$= \sum_{A=1}^{C} (-14A^3 + A^2(18C+27) - A(6C^2+18C+7) + (2C^3+3C^2-5C-6))/24$$
$$= C(C+1)(C+2)(3C-5)/48. \tag{13}$$

Finally, for the fourth summation we derive:

$$\sum_{A=1}^{C} \sum_{B=A}^{C} \sum_{X=B+1}^{C} \min(|X-A|, |X-B|)$$
$$= \sum_{A=1}^{C} \sum_{B=A}^{C} \sum_{X=B+1}^{C} (X-B) = \sum_{A=1}^{C} \sum_{B=A}^{C} (B^2 - B(2C+1) + C^2 + C)/2$$
$$= \sum_{A=1}^{C} (-A^3 + 3A^2(C+1) - A(3C^2+6C+2) + C(C^2+3C+2))/6$$
$$= (C-1)C(C+1)(C+2)/24. \tag{14}$$

If $A+B$ is even, then expression (1) is equal with the sum of expressions (3, 7, 12, 14), otherwise is equal to the sum of expressions (3, 9, 13, 14). That is:

$$\begin{array}{ll} C^2(5C^2-5)/24, & \text{if } A+B \text{ is even,} \\ C^2(5C^2-11)/24, & \text{if } A+B \text{ is odd.} \end{array} \tag{15}$$

The denominator is equal to the total number of ways that the head positions and the hit cylinder may be chosen. Evidently this number is C^3. Therefore, the average seek distance for answering a request is approximated by:

$$5C/24 \quad \text{either } A+B \text{ is even or odd.} \tag{16}$$

3. Analysis for intelligently positioned heads

In [4] a new scheduling policy for disk systems with two independent heads is outlined. More specifically, it is proposed to implement the following technique. When a request arrives, the closest head moves to service the request, while the other head does not remain idle but jockeys to take a position anticipating the cylinder number that the next request will hit. It is assumed, also, that this jockeying is performed at no cost.

Along those lines we examine the following method. When one head services a request at a cylinder number smaller than $C/2$ then the other head should lie somewhere between the hit cylinder and cylinder C. Otherwise, if the first head services a request at a cylinder number greater than $C/2$, then the other head should take a position somewhere between the first cylinder and the hit cylinder.

In an early work by Waters a conventional one-headed disk was examined [9]. It was estimated that if the head is randomly positioned on top of a cylinder and a random request arrives, then the mean head movement to answer the request is approximately $C/3$. Therefore, in our case we decide that if the first head points a cylinder A smaller than $C/2$, then the other head will be positioned on top of the cylinder numbered $A+\lfloor 2(C-A)/3 \rfloor$, otherwise, if A is greater than $C/2$, then the other head will be positioned on top of the cylinder numbered $\lfloor A/3 \rfloor$. In this way the two heads are not randomly positioned as in the previous section. Instead, the position of one head depends closely on the specific position of the other one. Figure 1 shows examples for two such occurrences.

The analysis for the expected seek distance traveled follows. As in the previous section, the seek distance traveled is equal to the ratio of the sum of distances divided by the number of all the possible

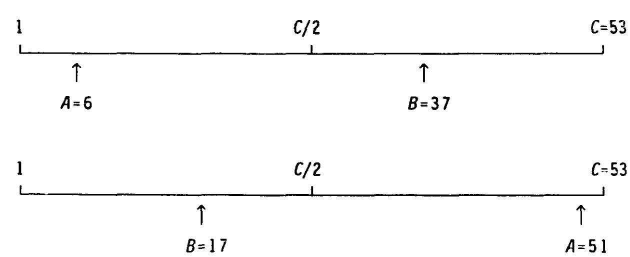

Fig. 1. Two occurrences of intelligent head positionings.

combinations of head positions and hit cylinders. The nominator is equal to the sum of eight double summations:

$$\sum_{A=1}^{\lfloor C/2\rfloor}\left(\sum_{X=1}^{A-1}(A-X)+\sum_{X=A+1}^{A+\lfloor(C-A)/3\rfloor}(X-A)+\sum_{X=A+\lfloor(C-A)/3\rfloor+1}^{A+\lfloor 2(C-A)/3\rfloor-1}\left(\left\lfloor\frac{2(C-A)}{3}\right\rfloor+A-X\right)\right.$$
$$\left.+\sum_{X=A+\lfloor 2(C-A)/3\rfloor+1}^{C}\left(X-\left\lfloor\frac{2(C-A)}{3}\right\rfloor-A\right)\right)$$
$$+\sum_{A=\lfloor C/2\rfloor+1}^{C}\left(\sum_{X=1}^{\lfloor A/3\rfloor-1}\left(\left\lfloor\frac{A}{3}\right\rfloor-X\right)+\sum_{X=\lfloor A/3\rfloor+1}^{\lfloor 2A/3\rfloor}\left(X-\left\lfloor\frac{A}{3}\right\rfloor\right)+\sum_{X=\lfloor 2A/3\rfloor+1}^{A-1}(A-X)\right.$$
$$\left.+\sum_{X=A+1}^{C}(X-A)\right). \tag{17}$$

It is difficult to simplify this relation because it contains many floor functions. Branching with all the possible combinations of A's and C's produces nine different results. We assume that the argument of every floor function is integer and derive an upper bound for the nominator. After some algebra we derive that the nominator equals:

$$\sum_{A=1}^{C/2}\left((A^2-A)/2+(C-A)(C-A+3)/18+(C-A)(C-A-3)/18\right.$$
$$\left.+(C-A)(C-A+3)/18\right)$$
$$+\sum_{A=C/2+1}^{C}\left((A^2-3A)/18+(A^2+3A)/18\right.$$
$$\left.+(A^2-3A)/18+(C-A)(C-A+1)/2\right) \tag{18}$$

$$=\sum_{A=1}^{C/2}(4A^2-2A(C+2)+C^2+C)/6+\sum_{A=C/2+1}^{C}(4A^2-2A(3C+2)+3C^2+3C)/6$$
$$=C(5C-8)/72+C(5C-8)/72=C(5C-8)/36. \tag{19}$$

The possible number of occurrences of head positionings and cylinder hits is, evidently, C^2. Therefore, the average seek distance for answering a request is approximated by:

$$5C/36. \tag{20}$$

Note that this figure is reported in [4] with no proof.

4. Comparison and discussion

It was estimated by Waters that the mean head movement to answer a randomly hit cylinder, if the head of a conventional one-headed disk system is also randomly positioned, is approximately $C/3$ [9]. In [6] it was proved that a disk system with two read/write heads at fixed distance between them reduces the seek distance traveled to (less than) 50% if the heads are optimally distanced. The optimal distance separating the two heads is equal to $C/2-1$ ($\lceil C/2-1 \rceil$ and $\lfloor C/2-1 \rfloor$) if C is even (odd). Therefore, we get that in this case the mean seek distance to answer a random request is approximately $C/6$.

In this paper a disk system with two independent heads was examined. If the heads are randomly positioned, then the average seek distance traveled to answer a random request is $5C/24$ approximately. If the heads are intelligently positioned according to the scheduling policy described, then the average seek distance traveled to answer a random request is $5C/36$. Therefore, we remark a considerable gain. Note that this study, as well as other ones [4,5,9] is based on the implicit assumption that the cylinders are hit equiprobably. Assuming other probability distributions other estimates for the traveled seek distances should be derived. However our results are useful as a first baseline.

These are some preliminary results. However, it is shown that disk systems with two autonomously driven heads operating under some scheduling policy, which may exploit the special disk characteristics, outperform disk systems with two heads at a fixed distance. Further study of new sophisticated scheduling algorithms together with data placement strategies by analysis and simulation is necessary. Another possible direction for further study is to accept that large files are partitioned and span many disk cylinders. Assuming a varying locality strength for each file more realistic results could be reached.

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