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Parallel data paths in two-headed disk systems☆

Athena Vakali, Yannis Manolopoulos*

Department of Informatics, Aristotle University, 54006 Thessaloniki, Greece

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Abstract

Analytic models on the expected seek distances for a set of k shadowed conventional disks with one read/write head per surface have been reported in the past. In the present paper we study the expected seek performance in the case of parallel disks having two heads per surface. Two-headed disks may have either two autonomous heads per surface or may have two heads moving concurrently at a fixed distance. The performance of both models is examined by assuming that successive seeks are either independent or dependent. Exact new formulae are derived for each of the aforementioned cases and generalized performance comparisons of all these disk configurations are made.

Keywords: Single-headed disks; Two-headed disks; Shadowed disks; Mirrored disks; Seek distance; Seek time; Performance evaluation

1. Introduction

Given the arrangement of data surfaces and read/write heads, the time required for a particular disk operation mainly involves the following actions [1]:

- move the appropriate head to the appropriate cylinder (*seek time*),
- wait for the required sector to rotate around to the location of the r/w head (*latency time*), and
- read the bytes from the disk surface (*block transfer time*).

With a variable speed disk the average latency varies according to the track being accessed. Also, the system manager has some control over rotational delays by arranging data on the recording surfaces using special placement techniques (e.g. hopscotching). On average, latency time equals half a revolution for the required data to appear at the r/w position. A typical rotational speed for a constantly rotating disk is 3600 rpm. This yields an average latency of 8.3 ms for any track. Recently rotational speed has increased to 7200 rpm [2], whereas the use of cache memory decreases the effect of latency time even further. In a similar manner, block transfer time is proportional to the rotational speed and depends on the amount of data, since one or more disk blocks may be transferred. Next, we concentrate on seeking.

Recently, there has been a considerable interest in shadowed (or mirrored) parallel disks, where all disks are identical and store the same data. This way, each disk may be viewed as a copy of the others. In such systems, reliability, fault tolerance and enhanced performance are achieved at the expense of storage space. Reading data is satisfied by accessing any of the disks since they all store exactly the same data. The choice of the disk to be accessed is made by applying the '*minimum distance*' policy, i.e. choose the disk on which the appropriate r/w head is closest to the requested cylinder. Writing new information must be satisfied by all disks since they all have to be identical copies. In [3] and [4] analytic models have been developed which study the performance behavior of seeking. More specifically, analytical expressions for the seek distance traveled are derived for the case of reads and writes as functions of the number of disks.

Disks with two heads per surface have been commercially available and may be categorized in two different models. According to the first category, the two heads are mounted on one moving arm and, thus, they move

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* email: {avakali,manolopo}@athena.auth.gr

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concurrently always being at a fixed distance from each other. Mathematical models for this kind of two-headed disk have been studied in [5] by adopting the first-come-first-served (FCFS) disk scheduling policy, in [6] by examining the SCAN policy, whereas in [7] simulation results have been reported for a number of scheduling policies. The common conclusion of all these works is that these systems behave almost twice as well as conventional one-headed systems in terms of the seek distance traveled. Previous work related to this kind of two-headed disk has also covered data placement techniques [8]. The second category comprises of disks with two autonomously moving heads/surface. Such systems have been first studied in [9], where a greedy algorithm has been proposed to optimally position the idle head on top of a specific cylinder in anticipation of future requests, while the other head is serving a request. In [10–13], also, estimates have been given for the average seek distance traveled in such systems.

In this paper we consider a set of $k \geq 2$ identical two-headed disks, where each disk has C cylinders, and we examine both kinds of two-headed systems. We assume, also, that single requests arrive uniformly distributed to all the cylinders. The structure of the remainder of the paper is as follows. In Section 2 one-headed shadowed disk systems are described. In Section 3 we study a system of parallel two-headed disks with autonomous heads assuming that either the successive seeks are independent (Section 3.1) or dependent on each other (Section 3.2). In Section 4 a similar analysis is carried out for a model of parallel two-headed disks with heads being always at a fixed distance, taken as half of the total number of cylinders C. Analysis is based on both assumptions, i.e. that successive seeks are independent (Section 4.1) or dependent on each other (Section 4.2). The expected seek distances traveled for reads and writes are calculated and the total expected seek distance is studied for different read/write ratios. Comparisons between all the aforementioned disk configurations are discussed. Finally, future work areas are suggested in Section 5.

2. Shadowed one-headed disks

In a parallel system with shadowed disks identical data are stored in all disks. In order to maintain such a system, certain considerations for both read and write requests must hold. For example, reading data is satisfied by accessing any of the disks since they all store exactly the same data. The choice of the disk which will be accessed is made by applying the 'minimum distance' policy, i.e. the disk on which the appropriate r/w head is closest to the required cylinder. Also, writing new data must be satisfied by all disks since they all have to be identical parallel copies.

The use of such parallel disk systems provides both reliability and fault tolerance. An immediate backup service is supported, while data are accessible whenever at least one disk is available. Since the disk choice is optimized, there is a certain reduction in expected seeks for reads, whereas seek performance for parallel writes will be at most the maximum of the seek distances instead of being their sum. Typically the time required to move the r/w heads is a function of the number of tracks over which the heads must travel. The function is not linear, e.g. if x ms are required to travel over one track, then moving over C tracks normally takes less than $x * C$ ms. Typical figures are 4 ms in the best case (for a single track movement), 28 ms in the worst case (innermost track to outermost track or vice versa) and 16 ms on average (e.g. for the HP C2200A disk [2]). It is proven in [3,4] that such parallel one-headed disk images reduce the access time for r/w requests, in general.

The model in [3,14], assuming that successive seeks are independent of each other, resulted in specific expressions for the expected seek distances traveled for both read and write requests:

$$\mathrm{E}[read] = \frac{C}{2k+1} \tag{1}$$

and

$$\mathrm{E}[write] = C(1 - I_k) \tag{2}$$

where

$$I_k = \begin{cases} \dfrac{2k}{2k+1} I_{k-1} & \text{if } k > 1 \\ 2/3 & \text{if } k = 1 \end{cases}$$

The total expected seek distance is calculated by using both read and write seek distances and by introducing different ratios between reads and writes respectively. Thus, the total expected seek distance traveled becomes:

$$\mathrm{E}[total] = r\mathrm{E}[read] + w\mathrm{E}[write] \tag{3}$$

where r is the read percentage and w is the write percentage, so that $r + w = 1$. Thus, the total expected seek distance in the case of a single-headed parallel

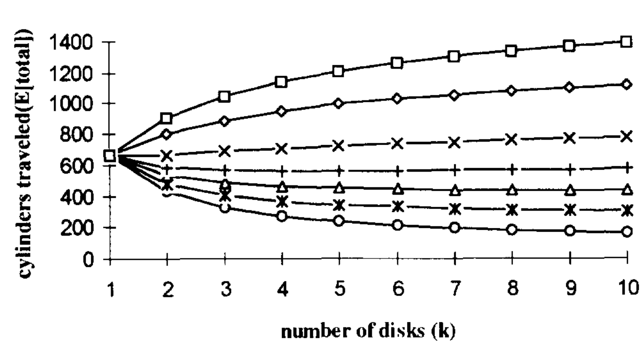

Fig. 1. Total expected seek distance as a function of k and r according to [3] ($C = 2000$). –O– $r = 0.95$; – ∗ – $r = 0.85$; –Δ– $r = 0.75$; – + – $r = 0.65$; – × – $r = 0.5$; –◇– $r = 0.25$; –□– $r = 0.05$.

disk set with independent seeks is given graphically in Fig. 1 as a function of the number of disks ($2 \leq k \leq 10$) and the read percentage $0.05 \leq r \leq 0.95$, whereas the number of cylinders is $C = 2000$.

In [4] a more refined model was developed by introducing Markov chains and dependency between successive seeks. The result was that the system will behave as if the number of parallel disks were reduced. The new formulae were produced for expected seek in both reads and writes:

$$\mathrm{E}[read] = \sum_{i=1}^{k} \pi_i \frac{C}{2i+1} \tag{4}$$

and

$$\mathrm{E}[write] = \sum_{i=1}^{k} \pi_i C(1 - I_i) \tag{5}$$

where π_i is the long-run proportion of the time the process spends in state i ($i = 1, 2, \ldots, k$):

$$\pi_i = \pi_1 \prod_{j=2}^{i} \frac{n_{j-1}}{n_j + w}$$

where:

$$\pi_1 = \left(\sum_{i=1}^{k} \prod_{j=2}^{i} \frac{n_{j-1}}{n_j + w} \right)^{-1}$$

and

$$n_j = \begin{cases} r & \text{if } j = 1 \\ r/i & \text{if } 1 < i < k \\ 0 & \text{if } j = k \end{cases}$$

Again, the total expected seek distance traveled is given by Equation (3). Fig. 2 gives graphically the total expected seek distance for the case of a single-headed parallel disk set with dependent seeks as a function of the parameters k and r. Here, $2 \leq k \leq 12$, $0.05 \leq r \leq 0.95$.

3. Parallel two-headed disks with autonomous heads

In this section, a system of parallel identical disks with two read/write heads per surface is considered. The two r/w heads are mounted on two separate arms and move autonomously and independent of each other. Thus, the two heads of a specific surface (let us call them: head A and B) can access any track of the surface in question. Reading and writing are satisfied as described in Section 2. Seek time may be approximated by the distance (number of cylinders) traveled by the heads, when the arm moves from the current cylinder to the requested one. Uniform distribution of requested cylinder positions is assumed. Although this is not the general case, it serves as a good starting point to estimate the expected seek distance. Data placement has a major impact on the seek distance and it is a further major study area.

Consider a system of k disks having two autonomous heads per surface. Thus, all the heads lie on at most $2 * k$ different cylinders in the k disks. Since each request refers to a certain cylinder, the requested cylinder imposes the use of $2 * k$ independent variables with the same distribution: i.e. k indpendent variables $(a_1, a_2, \ldots, a_k)$ for the distances of the head A from the requested cylinder for each disk and k independent variables $(b_1, b_2, \ldots, b_k)$ for the distances of the head B from the requested cylinder for each disk. We have C cylinders per disk in total, therefore there are C^2 unique seeks (C of size 0 and $2 * (C - i)$ of size $i = 1, 2, \ldots, C - 1$). Thus, the following relations hold:

$$P(a = i) = \frac{2(C-i)}{C^2} \qquad P(b = i) = \frac{2(C-i)}{C^2}$$

$$P(a \geq i) = \frac{(C-i)(C-i+1)}{C^2}$$

$$P(b \geq i) = \frac{(C-i)(C-i+1)}{C^2}$$

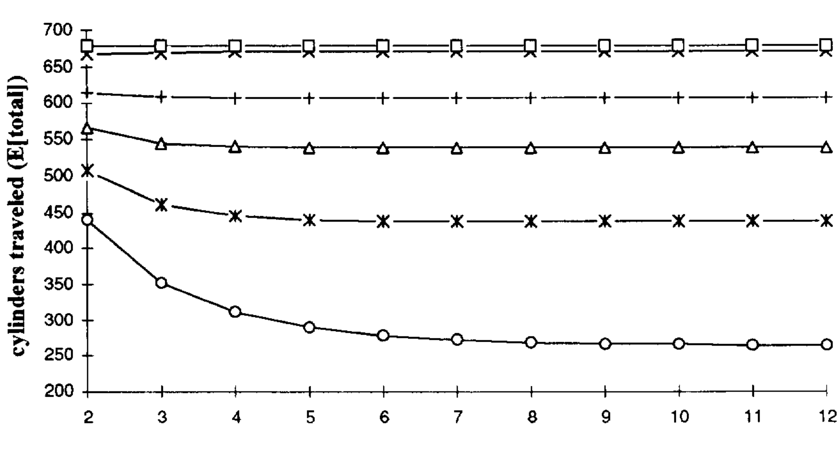

Fig. 2. Total expected seek distance as a function of k and r according to [4] ($C = 2000$). Key as in Fig. 1.

$$P(a > i) = \frac{(C-i)(C-i-1)}{C^2}$$

$$P(b > i) = \frac{(C-i)(C-i-1)}{C^2}$$

3.1. Expected distances for independent seeks

In this subsection we adopt the assumption of [3]. In case of a read, one of the disk heads satisfies the minimum distance property and is sufficient for serving the request. In other words we have that:

$$v_{r_i} = P[\min(a_1, \ldots, a_k, b_1, \ldots, b_k) \geq i]$$
$$= P(a_1 \geq i) \ldots P(a_k \geq i) P(b_1 \geq i) \ldots P(b_k \geq i)$$

Therefore, the resulting expected distance traveled for reads is (see the appendix for the analytical derivation):

$$\mathrm{E}[read] = \sum_{i=1}^{C-1} v_{r_i} \approx \frac{C}{4k+1} \tag{6}$$

In the case of a write, all the disks will execute the request. Thus, the expected seek distance for writes is: $\max[\min(a_1, b_1), \ldots, \min(a_k, b_k)]$. A new independent variable, v_{w_i}, is introduced. Similarly:

$$v_{w_i} = P\{\max[\min(a_1, b_1), \ldots, \min(a_k, b_k)] \geq i\}$$
$$= 1 - P\{\max[\min(a_1, b_1), \ldots, \min(a_k, b_k)] < i\}$$
$$= 1 - P[\min(a_1, b_1) < i] \ldots P[\min(a_k, b_k) < i]$$
$$= 1 - \{1 - P[\min(a_1, b_1) \geq i]\} \ldots$$
$$\times \{1 - P[\min(a_k, b_k) \geq i]\}$$
$$= 1 - \{1 - P(a_1 \geq i)P(b_1 \geq i)\} \ldots$$
$$\times \{1 - P(a_k \geq i)P(b_k \geq i)\}$$

Therefore, we have (see the Appendix for details):

$$\mathrm{E}[write] = \sum_{i=1}^{C-1} v_{w_i} \sim C(1 - I_k) \tag{7}$$

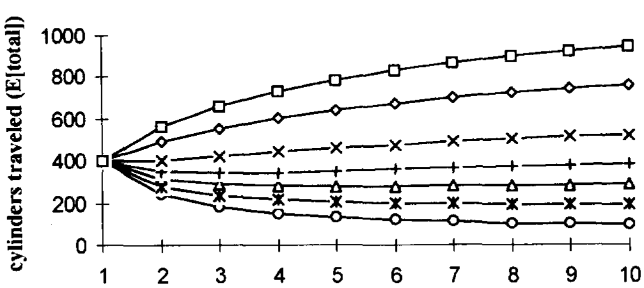

Fig. 3. Total expected seek distance as a function of k and r for two-headed disks with autonomous heads assuming independence between seeks ($C = 2000$). Key as in Fig. 1.

where

$$I_k = \begin{cases} \frac{4k}{4k+1} I_{k-1} & \text{if } k > 1 \\ 4/5 & \text{if } k = 1 \end{cases}$$

The graph represented in Fig. 3 displays the total expected seek distance traveled for the same parameters and ranges as in the previous two figures. The expected seek distance for reads (i.e. for $r = 0.95$) appears to be a decreasing monotonic function, whereas the expected seek distance for writes (i.e. for $r = 0.05$), shows an increasing behavior. As a common starting point of the expected seeks we consider the $k = 1$, which corresponds to the case of the usual two-headed disk systems operating under the FCFS policy. Comparing the present results of the ones of [3] (as they are displayed graphically in Fig. 1), it is shown that there is a performance improvement in the total expected seek distance, which varies from 32% (when $r = 0.05$ and $k = 10$) up to 45% (for the case of $r = 0.95$ and $k = 3$).

3.2. Expected distances for dependent seeks

In the previous section, the seek distances were assumed to be independent as in [3]. The need for each write request to be served by all disks imposes a dependency between successive seeks, since after a write, a set of k heads will be in identical positions [4]. Thus, in order to serve the request following a write, the choice is made out of at most $k + 1$ (and not $2 * k$) different cylinders. A Markov chain can be modeled to depict the actual number of different cylinder positions with state-space $\{2, \ldots, 2k\}$, whereas the transition function is formed by considering all the possible fluctuations of the number of different cylinders occupied by r/w heads. There are three cases concerning the fluctuation of this number:

1. Due to a read, the number remains the same. This happens when an arriving read request refers to a cylinder 'occupied' already by a head. We denote this probability by $p(i, i)$, where $i \in [2..2k]$. This probability is equal to:

   $$s_i = p(i, i) = \frac{i-1}{i} r \qquad \text{where } 2 < i < 2k$$

   This is explained by the fact that there are i different positions, whereas $i - 1$ positions correspond to $i - 1$ different heads on top of them and one position has $2k - i + 1$ heads on top of it, since in order to remain in a state i one of the $i - 1$ heads must move. Notice that the boundary values are: $s_2 = 0$ and $s_{2k} = r$.
2. Due to a read, the number is increased by 1. The case arises when an arriving read request refers to a cylinder not 'occupied' already by r/w heads. We denote this probability by $p(i, i+1)$, where $i \in [2..2k-1]$.

This probability may be calculated by:

$$n_i = p(i, i+1) = \frac{1}{i} r \qquad \text{where } 2 < i < 2k$$

since one of the $2k - i + 1$ heads, which lie in one identical position must move. Notice, also, that the boundary values are: $n_2 = r$ and $n_{2k} = 0$.

3. Due to a write, the number changes from i to j, where $i \in [2..2k], j \in [2..k+1]$ and $(i \geq j)$. The range of j is $[2..k+1]$, since an arriving write request forces k heads to move to identical positions, while the other heads may lie in $1, 2, .., k$ different positions. In order to calculate the probability $p(i,j)$ we have to examine two subcases:
   - $i \in [2..k+1]$. In such a case: $j \in [2..i]$. Then:

$$p(i,i) + p(i,i+1) + \sum_{j=2}^{i} p(i,j) = 1$$

   By assuming that all $i - 1$ possibilities for the index j are equally likely, we have:

$$wa_i = p(i,j) = \frac{1}{i-1} w \qquad \text{where } i > 2$$

   with boundary value $wa_2 = w$.
   - $i \in [k+2..2k]$. Therefore: $j \in [i-k+1..k+1]$. Then we have:

$$p(i,i) + p(i,i+1) + \sum_{j=i-k+1}^{k+1} p(i,j) = 1$$

   By assuming that all $2k - i + 1$ possibilities for j are equally likely, we have:

$$wb_i = p(i,j) = \frac{1}{2k-(i-1)} w \qquad \text{where } i < 2k$$

   with boundary value $wb_{2k} = w$.

Introducing the long-run proportion of the time the process spends in state i we have:

- for the case $i \in [2..k+1]$:

$$\pi_i = \pi_i s_i + \pi_{i-1} n_{i-1} + \sum_{j=i}^{k+1} \pi_j wa_j + \sum_{j=k+2}^{k+i-1} \pi_j wb_j \Rightarrow$$

$$\pi_{i-1} = \pi_i \frac{n_i + (i-2)wa_i}{n_{i-1}} - \sum_{j=i+1}^{k+1} \pi_j \frac{wa_j}{n_{i-1}} - \sum_{j=k+2}^{k+i-1} \pi_j \frac{wb_j}{n_{i-1}}$$

where:

$$\pi_{i-1} = \pi_{k+1} f_{i-1} \qquad \text{for } 2 < i \leq k+1$$

with $f_{k+1} = 1$ and

$$f_{i-1} = f_i \frac{n_i + (i-2)wa_i}{n_{i-1}} - \sum_{j=i+1}^{k+1} f_j \frac{wa_j}{n_{i-1}} - \sum_{j=k+2}^{k+i-1} \frac{wb_j}{n_{i-1}} \prod_{l=k+2}^{j} g_l$$

- whereas in the case $i \in [k+2..2k]$:

$$\pi_i = \pi_i s_i + \pi_{i-1} n_{i-1} \Rightarrow$$

$$\pi_i = \pi_{i-1} \frac{n_{i-1}}{1 - s_i} \Rightarrow$$

$$\pi_i = \pi_{k+1} \prod_{j=k+2}^{i} g_j \qquad \text{for } k+2 \leq i \leq 2k$$

where

$$g_j = \frac{n_{j-1}}{1 - s_j} \qquad \text{for } k+2 \leq j \leq 2k$$

Since $\Sigma_{i=2}^{2k} \pi_i = 1$ we have that:

$$\pi_{k+1} = \left(1 + \sum_{i=2}^{k} f_i + \sum_{l=k+2}^{2k} \prod_{l=k+2}^{i} g_l\right)^{-1}$$

Figure 4 depicts the Markov chain describing a system with $k = 3$ disks. Evidently, the number of states, i, equals 5, i.e. from 2 to 6. The corresponding probabilities for reads and writes for each specific state are also depicted in the figure.

By using these formulae and the formulae for expected read and write seek found in Section 3.1, the new expected seek for reads and writes will be (respectively):

$$\mathrm{E}[read] = \sum_{i=2}^{2k} \pi_i \frac{C}{4i+1} \tag{8}$$

and

$$\mathrm{E}[write] = \sum_{i=2}^{2k} \pi_i C(1 - I_i) \tag{9}$$

The main observation is that by considering the

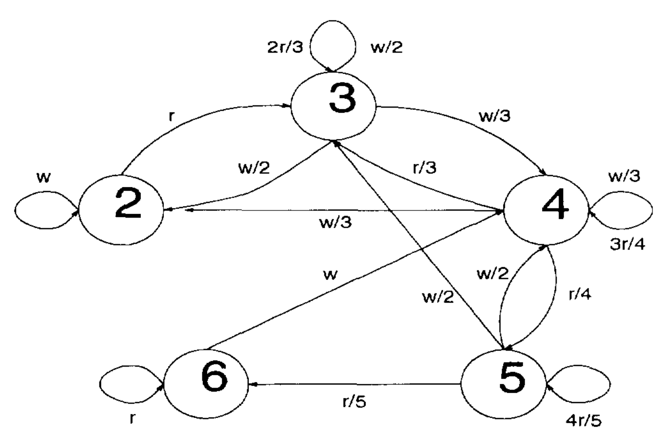

Fig. 4. Markov chain describing a system with $k = 3$ disks.

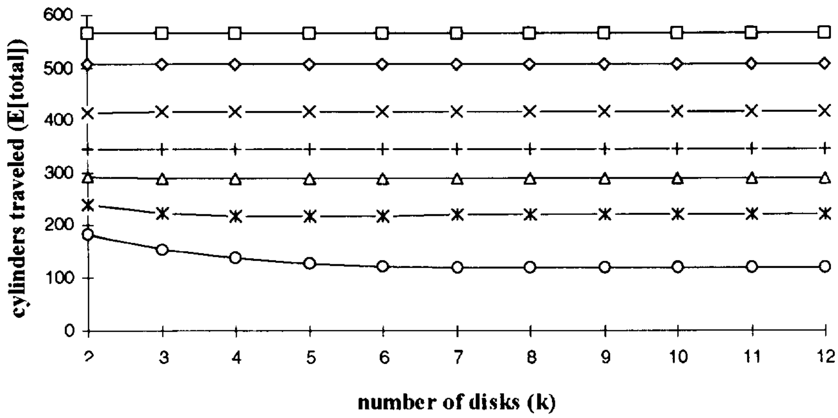

Fig. 5. Total expected seek distance as a function of k and r for two-headed disks with autonomous heads assuming dependency between seeks ($C = 2000$). Key as Fig. 1.

dependency between successive seeks, the new model shows a worse (respectively, better) behavior in the case of the expected seek for reads (respectively, writes), in comparison to the results of the model introduced in Section 3.1. Using these new equations (8) and (9), the overall expected seek distance $E[total] = rE[read] + wE[write]$ is represented in Fig. 5, where $0.05 \leq r \leq 0.95$ and $C = 2000$ cylinders. By comparing these results with those of the model in [4] (as they are shown in Fig. 2), we remark that the two-headed disk model outperforms the conventional one-headed one. For some cases, the present two-headed model seems to behave more than twice as well as the one-headed disk model. As depicted in Fig. 6 the improvement rate varies between 16% (when $r = 0.05$ and $k = 10$) and 59% (when $r = 0.95$ and $k = 2$). As indicated in Fig. 6 there is more than a 50% gain when the read ratio is $r > 0.6$, whereas for smaller values of r there is also a considerable gain percentage between the two models.

4. Parallel two-headed disks with fixed heads

In this section, a system of k parallel disks is considered again. In each disk there are two r/w heads per surface which do not move independently, but they are mounted on the arm remaining always at a fixed distance and move concurrently in the same direction all the time. None of the two heads can move outside the disk surface. This model has been studied before (e.g. [5,6]) and the optimal fixed distance between the two heads is found to be half of the data band. Thus, the position of the second head B is found by adding $d = C/2$ to the cylinder position of the first head A. This fact imposes a division of the data band in two intervals: $[1, d]$ and $[d + 1, C]$. The area of the first interval $[1, d]$ is served only by the first head A, while the area of the $[d + 1, C]$ interval is scanned only by the second head B. Each request refers to a certain cylinder and in order to be served, the interval to which it belongs (i.e. which head will satisfy it) must be identified. Thus, one head moves to the requested location while the other head moves simultaneously in the same direction but always at a fixed distance d.

For each request referring to the interval $[1, d]$, there are k independent variables to represent the distance of head A (in all of the k parallel disks) from the requested cylinder. In total, each head can access only up to d different cylinders, so there are d^2 unique seek distances for each head (d of size $0, 2 * (d - i)$ of size

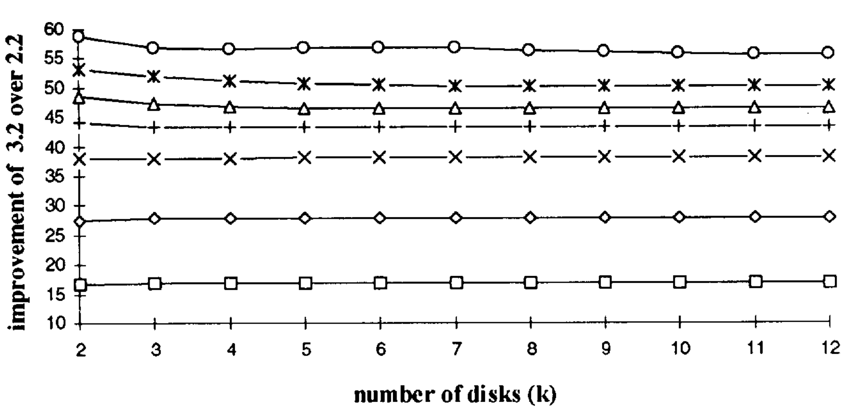

Fig. 6. Improvement rate (%) of two-headed disks with autonomous heads over the one-headed disk system ($C = 2000$). Key as Fig. 1.

$i = 1, 2, \ldots, d-1$). Thus the following relations hold:

$$P(a = i) = \frac{2(d-i)}{d^2}$$

$$P(a \geq i) = \frac{(d-i)(d-i+1)}{d^2}$$

$$P(a < i) = \frac{(i-1)(2d-i)}{d^2}$$

4.1. Expected distances for independent seeks

In the case of a r/w request, first the interval of the requested cylinder is identified (i.e. it is checked whether the requested cylinder is $\leq$ or $> d$). One of the heads will serve the request from either of the k disks. The disk to serve the request will be identified by applying the minimum distance policy for the k disks, i.e. the disk on which the head is closer to the requested cylinder will satisfy the request. In a similar manner as in Section 3.1, we have that:

$$v_{r_i} = P[\min(a_1, \ldots, a_k) \geq i] = P(a_1 \geq i) \ldots P(a_k \geq i)$$

Therefore, we may calculate the expected seek distance for reads as follows (see the appendix for details):

$$\mathrm{E}[read] = \sum_{i=1}^{C-1} v_{r_i} \approx \frac{d}{2k+1} \quad (10)$$

In the case of a write, all the disks have to execute the request. Therefore, all the heads of the respective interval will move to serve a write operation. Thus, the expected seek distance for writes is $\max(a_1, a_2, \ldots, a_k)$. Similarly,

$$v_{w_i} = P[\max(a_1, \ldots, a_k) \geq i] = 1 - P[\max(a_1, \ldots, a_k) < i] = 1 - P(a_1 < i) \ldots P(a_k < i)$$

Thus for the expected distance traveled for writes we have (see the appendix for details):

$$E[write] = \sum_{i=1}^{d-1} v_{w_i} \approx d(1 - I_k) \quad (11)$$

where

$$I_k = \begin{cases} \frac{2k}{2k+1} I_{k-1} & \text{if } k > 1 \\ 2/3 & \text{if } k = 1 \end{cases}$$

As in the previous section, the total expected seek distance traveled is calculated by considering both read and write expected seek distances and by using different ratios between read and writes respectively, i.e. as in Equation (3). The graph represented in Fig. 7 illustrates the total expected seek distance under the same parameters as in Fig. 3. Again, the expected seek distance for reads appears to be a decreasing monotonic function, whereas the expected seek distance for writes shows an increasing behavior. As depicted in Figs. 1 and 7, the total expected seek distance according to the new model shows a 50% improvement when compared to that of the model presented in [3]. Comparing this model with the model of Section 3.1 we notice that there is almost a 10% to 27% improvement, as r varies from 0.95 to 0.05. More specifically the rates vary between 10% (when $k = 2$ and $r = 0.95$) and 27% (when $k = 10$ and $r = 0.05$). Assuming that the two heads remain at a fixed distance from each other (taken in the present paper as half of the data band), and that successive seeks are independent, the two-headed disk model seems to behave better, especially when serving the write request. There needs to be further investigation to establish the effect on performance if the head separation distance is different from half of the data band.

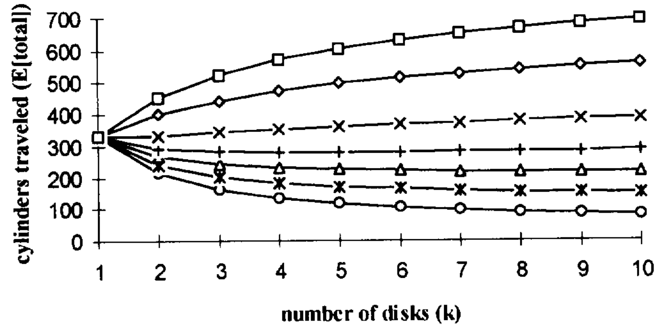

Fig. 7. Total expected seek distance as a function of k and r for two-headed disks with fixed heads assuming independence between seeks ($C = 2000$). Key as Fig. 1.

4.2. Expected distances for dependent seeks

As discussed in Section 3.2, the seeks are not really independent of each other because writing must be served by all disks. Suppose that an arriving write request refers to any of the two intervals $[1, d]$ or $[d+1, C]$. Every write causes k heads to move to an identical cylinder position x (suppose $x \in [1, d]$), so that the other k heads will lie on top of one identical cylinder too, namely the $(x+d)$-th cylinder. Therefore, there is only one position in $[1, d]$ and only one position in $[d+1, C]$ from which to choose in case the next request is a read. The read might increase the number of different cylinder positions by 1, or it might not change it. So all the possible fluctuations of the number of different cylinder positions in either interval $[1, d]$ or $[d+1, C]$ may be categorized in three cases, i.e.:

1. Due to a read, the number of 'occupied' cylinder positions remains the same, if the arriving read request

refers to an already occupied cylinder. We denote this probability by $p(i, i)$, where $i \in [1..k]$. This probability is equal to:

$$s_i = p(i, i) = \frac{i-1}{i} r$$

with boundary values $s_1 = 0$ and $s_k = r$.

2. Due to a read, the number of 'occupied' cylinder positions is increased by 1, if the read request arrives for a cylinder not 'occupied' by the r/w heads. We denote this probability by $p(i, i+1)$, where $i \in [1..k]$. This probability may be calculated by:

$$n_i = p(i, i+1) = \frac{1}{i} r$$

with boundary values $n_1 = r$ and $n_k = 0$.

3. Due to a write, the number of 'occupied' cylinder positions becomes 1 in either interval. We denote this probability by $p(i, 1)$ which is equal to:

$$w_i = p(i, 1) = w = 1 - r$$

A Markov chain is considered with state space $\{1, 2, \ldots, k\}$ in either interval and by introducing the long-run proportion of the time the process spends in states $i = 1, 2, \ldots, k$ we have:

$$s_i + n_i + w_i = 1$$

Introducing the long-run proportion π_i as in Section 3.2 we have:

$$\pi_i = \pi_i s_i + \pi_{i-1} n_{i-1} \Rightarrow$$

$$\pi_i = \pi_{i-1} \frac{n_{i-1}}{1 - s_i} \Rightarrow$$

$$\pi_i = \pi_{i-1} \frac{n_{i-1}}{n_i + w}$$

Therefore:

$$\pi_i = \pi_1 \prod_{j=2}^{i} \frac{n_{j-1}}{n_j + w}$$

Since $\Sigma_{i=1}^{k} \pi_i = 1$ we have:

$$\pi_1 = \left(\sum_{i=1}^{k} \prod_{j=2}^{i} f_j \right)^{-1}$$

where

$$f_j = \frac{n_{j-1}}{n_j + w}$$

These formulae in accordance with the formulae for the expected read and write seeks found in Section 4.1 result in:

$$\mathrm{E}[read] = \sum_{i=1}^{k} \pi_i \frac{d}{2i+1} \tag{12}$$

and

$$\mathrm{E}[write] = \sum_{i=1}^{k} \pi_i d(1 - I_i) \tag{13}$$

where

$$I_k = \begin{cases} \frac{2k}{2k+1} I_{k-1} & \text{if } k > 1 \\ 2/3 & \text{if } k = 1 \end{cases}$$

The main observation is that in the present case with dependency between seeks, a worse behavior is derived for the reads, while writes behave better, in comparison with the model of Section 4.1. Using these new formulae (with the same parameters in Fig. 5) the overall expected seek for the present model is represented in Fig. 8, which displays the 50% improvement compared to the one-headed model of [4]. The two-headed model of Section 3.2 compared to the present model shows an improvement in the read performances which reaches a 33% gain (for $k = 12$ and $r = 0.05$). Write performances of the model of Section 3.2 seems to worsen by almost 42% (for $k = 12$ and $r = 0.05$). In Fig. 9 the variations of the two models are shown graphically as percentages.

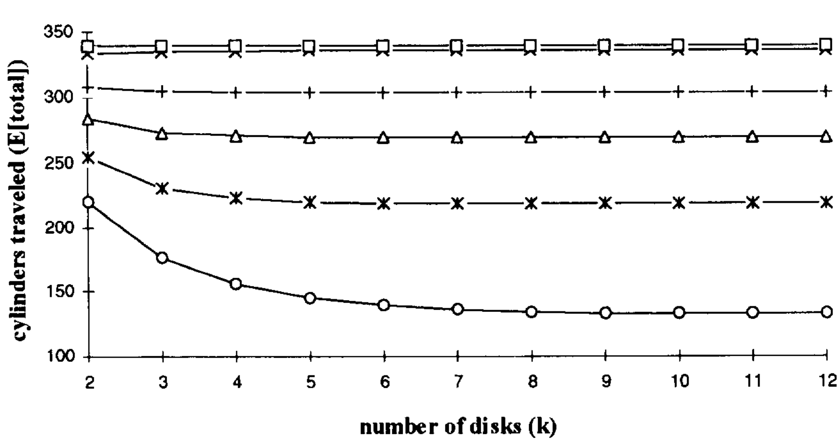

Fig. 8. Total expected seek distance as a function of k and r for two-headed disks with fixed heads assuming dependency between seeks ($C = 2000$). Key as Fig. 1.

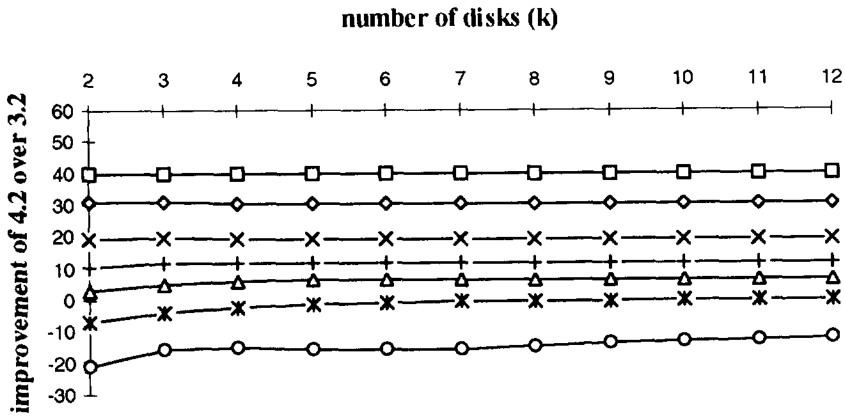

Fig. 9. Improvement rate (%) of two-headed disks with fixed heads over the two-headed disks with autonomous heads ($C = 2000$). Key as Fig. 1.

Thus, the present fixed-headed model behaves better than the Section 3.2 model in the case of writes, while the autonomous two-headed model of Section 3.2 outperforms this section's model in reading.

5. Epilog

Disk shadowing is a storage method which has been examined for conventional single-headed disk systems. This method provides enhanced fault tolerance, reliability and performance improvement at the expense of storage space, since the same data are kept in a number of parallel identical disks.

In the present report, analysis of the expected seek distances traveled for read and write requests was carried out by examining two-headed disks operating under the FCFS disk scheduling policy. Two cases of two-headed disk were examined. The first model has two heads which move autonomously, while in the second model the two heads are always separated by a fixed number of tracks (equal to half of the number of disk tracks). Each of the two models was studied under independence or dependency between successive seeks. Comparisons to previous models were made.

- The two-headed model with autonomous head movement shows a remarkable improvement compared to the one-headed model, with improvement rates reaching 45% (e.g. for the case of $k = 3$ and $r = 0.95$).
- The two-headed model with fixed heads out-performs the conventional one-headed disk system with a 50% improvement rate in cylinder distance traveling.

Concerning the two cases of two-headed models, the dependency between seeks is the crucial factor influencing the different models' performance.

- In the case of dependent seeks, the fixed two-headed model shows improvement in writes, while reads are worsening. For example, in case of $k = 12$ and $r = 0.05$ there is a 40% improvement in the total expected seek, whereas in case of $k = 2$ and $r = 0.95$ there is a 20% worsening rate.
- In the case of independent seeks, the autonomous two-headed model behaves better than the fixed two-headed model for any case (e.g. there is an 11% for $k = 2$ and $r = 0.95$ and a 26.6% for $k = 10$ and $r = 0.05$).

Under both assumptions, the performance improvement in terms of distance traveled for the case of two-headed disk systems over the conventional one-headed was proven to be substantial.

Further study would examine different disk scheduling policies which might be applied (e.g. SCAN, shortest-seek-time-first (SSTF)), derive estimates for the seek time (as opposed to the seek distance traveled) in modern non-linear disks, and/or new data placement techniques which might influence the system performance.

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Appendix A

Here, we prove the expressions for the expected distances E[*read*] and E[*write*], which appeared in Subsections 3.1 and 4.1 in the case of independent seeks for parallel two-headed disks with autonomous and fixed heads, respectively.

A.1. Parallel two-headed disks with autonomous heads

A.1.1. Proof of expression (6)

$$E[read] = \sum_{i=1}^{C-1} v_{r_i}$$

$$= \sum_{i=1}^{C-1} P(a_1 \geq i) \ldots P(a_k \geq i) P(b_1 \geq i) \ldots \times P(b_k \geq i)$$

$$= \sum_{i=1}^{C-1} \left(\frac{(C-i)(C-i+1)}{C^2} \right)^{2k}$$

$$\approx \frac{1}{C^{4k}} \sum_{i=1}^{C-1} (C-i)^{4k}$$

$$= C \sum_{i=1}^{C-1} \frac{1}{C} \left(1 - \frac{i}{C}\right)^{4k}$$

The sum of the latter expression could be calculated by the use of the Euler-MacLaurin formula but we use instead the Riemann's sum since the error term is insignificant and results in a good approximation. So the latter expression is taken as the Riemann's sum for the integral:

$$\int_0^1 \left(1 - \frac{i}{C}\right)^{4k} = \frac{1}{4k+1}$$

Thus, by replacing this result to the latter expression the expected seek for reads is:

$$\mathrm{E}[read] \approx \frac{C}{4k+1} \tag{6}$$

A.1.2. Proof of expression (7)

$$\mathrm{E}[write] = \sum_{i=1}^{C-1} v_{w_i}$$

$$= \sum_{i=1}^{C-1} 1 - \{1 - P(a_1 \geq i) P(b_1 \geq i)\} \ldots \times \{1 - P(a_k \geq i) P(b_k \geq i)\}$$

$$= \sum_{i=1}^{C-1} 1 - \left(1 - \frac{(C-i)^2 (C-i-1)^2}{C^4}\right)^k$$

$$\approx \sum_{i=1}^{C-1} 1 - \left(1 - \frac{(C-i)^4}{C^4}\right)^k$$

$$= \sum_{i=1}^{C-1} 1 - \left(1 - \left(1 - \frac{i}{C}\right)^4\right)^k$$

This sum is elaborated by using again the Riemann's sum for the integral:

$$I_k = \int_0^1 x^k (2-x)^k (2 - 2x + x^2)^k \, dx$$

and by replacing $u = 1 - x$ and $u = \sin v$. Thus, finally we derive that the expected seek for writes is:

$$\mathrm{E}[write] \approx C(1 - I_k) \tag{7}$$

where $I_k = 4k/4k + 1I_{k-1}$ and $I_1 = 4/5$.

A.2. Parallel two-headed disks with fixed heads

A.2.1. Proof of expression (10)

$$\mathrm{E}[read] = \sum_{i=1}^{d-1} v_{r_i}$$

$$= \sum_{i=1}^{d-1} P(\min(a_1, \ldots, a_k) \geq i)$$

$$= \sum_{i=1}^{d-1} P(a_1 \geq i) \ldots P(a_k \geq i)$$

$$= \sum_{i=1}^{d-1} \left(\frac{(d-i)(d-i+1)}{d^2} \right)^k$$

$$\approx \frac{1}{d^{2k}} \sum_{i=1}^{d-1} (d-i)^{2k}$$

$$= d \sum_{i=1}^{d-1} \frac{1}{d} \left(1 - \frac{i}{d}\right)^{2k}$$

The sum of the latter expression is the Riemann's sum for the integral:

$$\int_0^1 \left(1 - \frac{i}{d}\right)^{2k} = \frac{1}{2k+1}$$

Thus, by replacing this result to the latter expression the expected seek for reads is:

$$\mathrm{E}[read] = \frac{d}{2k+1} \tag{10}$$

A.2.2. Proof of expression (11)

$$\mathrm{E}[write] = \sum_{i=1}^{d-1} v_{w_i}$$

$$= \sum_{i=1}^{d-1} 1 - \frac{(i-1)(2d-i)}{d^2} \cdots \frac{(i-1)(2d-i)}{d^2}$$

$$= \sum_{i=1}^{d-1} 1 - \left(\frac{(i-1)(2d-i)}{d^2} \right)^k$$

$$\approx \sum_{i=1}^{d-1} 1 - \left(\frac{i-1}{d}\right)^k \left(\frac{2d-i}{d}\right)^k$$

$$= (d-1) - d \sum_{i=1}^{d-1} \frac{1}{d} \left(\frac{i-1}{d}\right)^k \left(2 - \frac{i}{d}\right)^k$$

This sum is elaborated by using the Riemann's sum again. For large values of C (and evidently for large values of d), finally we derive that the expected seek distance for writes is approximated by:

$$E[write] \approx d(1 - I_k) \tag{11}$$

where $I_k = 2k/2k + 1I_{k-1}$ and $I_1 = 2/3$.

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