# Reverse Nearest Neighbor Queries for Dynamic Databases * 

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#### Abstract

In this paper we propose an algorithm for answering reverse nearest neighbor (RNN) queries, a problem formulated only recently. This class of queries is strongly related to that of nearest neighbor (NN) queries, although the two are not necessarily complementary. Unlike nearest neighbor queries, RNN queries find the set of database points that have the query point as the nearest neighbor. There is no other proposal we are aware of, that provides an algorithmic approach to answer RNN queries. The earlier approach for RNN queries ([KM99]) is based on the pre-computation of neighborhood information that is organized in terms of auxiliary data structures. It can be argued that the precomputation of the RNN information for all points in the database can be too restrictive. In the case of dynamic databases, insert and update operations are expensive and can lead to modifications of large parts of the auxiliary data structures. Also, answers to RNN queries for a set of data points depend on the number of dimensions taken in considerations when initializing the data structures. We propose an algorithmic approach that is flexible enough to support a larger class of RNN queries, and, in order to support them, we also extend the current method of nearest neighbor search to that of conditional nearest neighbor.


## 1 Introduction

The problem of answering Reverse Nearest Neighbor (RNN) queries has been formally defined only recently by [KM99]. It is a complimentary problem to that of finding a Nearest Neighbor (NN) to a query point, studied in the past few years [RKV95, KSF + 96, SR99]. While NN queries find the point in the database that

[^0]is the closest to a given query point according to a specified distance metric, RNN queries find the set of database points that have the query point as the nearest neighbor. The complexity of the RNN problem arises from the fact that the NN/RNN relationship is asymmetric. That is, the fact that a query point $q$ has a data point $p$ as its nearest neighbor, does not imply that $p$ 's nearest neighbor is $q$ and therefore does not imply that $q$ 's RNN is the data point $p$. For example, let three points $a, b, c$ be placed on a straight line in this order, such that the nearest neighbor of $a$ is $b$ and the nearest neighbor of $b$ is $c$. Note that in this case, although the nearest neighbor of $a$ is $b, a$ has no reverse nearest neighbor. In this paper we present an efficient solution for RNN queries for 2-dimensional data, although our approach can be extended to data sets of higher dimensions. For 2-dimensional data, there is a wide range of applications that can benefit from efficiently implementing RNN queries.

Example: Consider a gas vendor company whose marketing decision is to expand by opening a new gas station in the same city. Given several choices for its new location, the company's strategy is to pick the location that can attract customers from other gas stations but, if possible, without negatively impacting its existing gas stations. Having a database that stores the location of all gas stations in the city together with additional information on businesses, an approach is to create a view that selects only the information on gas stations, and query this view for the gas stations that would be geographically closer to the new possible locations. In this case, an RNN query would return the set of gas stations that would be the most affected by a competitor in the given location (the query point). The company can also easily analyze the effects of competing with different companies by looking at the RNN queries of already existing locations.

In this paper we develop an algorithmic solution to the RNN problem that uses a generic index structure, which, in addition to searches for reverse nearest neighbors, can be used to answer any of the standard queries. The presentation is organized as follows. We
first present (Section 2) previously developed methods, compare them with our approach and summarize our goals in Section 3. In Section 3.1 we present the underlying ideas behind the algorithmic solution we propose for the problem of RNN queries. The algorithm is based on existing solutions for nearest neighbor queries, which in Section 3.3 we extend for the purpose of RNN queries to conditional nearest neighbor queries. We further present the details of the RNN algorithm (Section 3.4) and we conclude with a summary of possible extensions of our work in Section 4.

## 2 Background and Motivation

A solution for answering RNN queries for dynamic databases was recently proposed in [KM99]. This solution is based on storing a list of all the data points and their corresponding nearest neighbors in an NNtree, and the nearest neighborhood distance circles (for 2-dimensional data) in an RNN-tree. RNN queries become point enclosure queries over the RNN-tree. The use of an RNN tree to store information on reverse nearest neighbor may however be too rigid for querying the database. As an example, let a relational database contain tuples with multiple attributes. A tuple would then become a data point in the multidimensional space, but usually not all the available information is necessary to answer a given query. If we revisit the gas station example presented earlier, let the database actually contain tuples with more than the two attributes used to define the point as a geographical location. Assume tuples also include specific data on the type of service they offer. One dimension can therefore store the levels of service as four choices: 0) gas only, 1) mini market, 2) automatic car-wash and 3) hand car-wash. For the purpose of the facility location query this attribute represents information that is not significant and it should be ignored. The data set then is actually projected on only one plane (spanned by the two geographical direction axes). In general, different queries applied to a data set should be able to project the data points on the desired dimensions, and not be restricted to a given set or subset of attributes. So far, there has been no proposal for answering RNN queries in such a flexible manner. Since the set of nearest and reverse nearest neighbors to a query point may differ depending on how many dimensions are considered for the same data set,a data structure constructed specifically for answering RNN queries is too rigid. Consider a query point $q$ and the data set $\left\{p_{1}, p_{2}, p_{3}\right\}$ illustrated in Figure 1. If the query $\mathrm{RNN}(\mathrm{q})$ is defined over all three dimensions ( $X Y Z$ ), then the reverse nearest neighbor of $q$ is $p_{1}$. However, if the queried space is restricted to only two dimensions $(X Y)$, then RNN(q) will return data point $p_{2}$. We believe that to accommodate RNN queries spanning over a variable
subset of dimensions, an algorithmic approach is more appropriate.


Figure 1: Example of varying nearest neighbor set depending on number of dimensions

In this paper we consider dynamic databases, that undergo inserts and deletes of data points due to user updates. In [KM99] the update and query of dynamic databases is much more expensive than in the static case since insert and delete operations over the database lead to the computation of changes and to the modification of the NN-tree and RNN-tree indexing structures. In contrast, we are interested in RNN queries in the context of data warehouses used for decision support, where updates are frequent and I/O performance is important. As underlying data sources get updated frequently, the views in a data warehouse should reflect these changes and answer to user queries based on data that must be as up-to-date as possible. The update method therefore must be both simple and efficient. In this case, an algorithm based on continuously maintaining a list of the data points and their nearest neighbors is inefficient, since both inserting and deleting an element lead to the partial recomputation of the list. For the type of decision support queries we are interested in, combining the information on the state of the database with auxiliary information on pre-computed partial answers overloads the maintenance task of the view itself.

## 3 Our Solution

In this paper, we present our solution for answering RNN queries in a two-dimensional space. First, we will introduce the observations that lead to the proposed technique, then give a simple algorithm for finding the reverse nearest neighbor RNN(q) to a query point q. As we will show, our method is based on already
existing algorithms for answering nearest neighbor queries. Applications that use our method can benefit from the following advantages:

- Our goal is to integrate RNN queries in the framework of already existing database and access structures. The approach we are using is therefore algorithmic and independent of data structures constructed specifically for a set of such queries.
- Updates to the database only impact performance in the sense that they affect the size of the Rtree used for indexing the data set for range and spatial join queries. In the case of insert and delete operations over the database, no computational or storage cost will be imposed specifically to facilitate RNN queries. However, we assume that NN queries are supported in such systems.
- No additional data structures are necessary, and therefore the space requirement does not increase. The algorithm we propose is based on using the underlying indexing data structure (R-tree), also necessary to answer NN queries. We assume that a spatial data structure is necessary for many applications that require the ability to answer range queries, topological queries or nearest neighbor queries.
- The solution we present facilitates "what if" queries, based on RNN(q) queries where $q$ is not constrained to be one of the points in the data set. This class of queries are especially useful for decision making applications such as those involving summarization and interpretation of data in data warehouse views.


### 3.1 Preliminaries

In this paper we assume that the distance between two points $p_{1}=\left(p_{1_{1}}, p_{1_{2}}, \cdots p_{1_{n}}\right)$ and $p_{2}=\left(p_{2_{1}}, p_{2_{2}}, \cdots p_{2_{n}}\right)$ is the Euclidean distance: $\operatorname{dist}\left(p_{1} p_{2}\right)^{2}=\sum_{i=n}^{i=1} \mid p_{1_{i}}-$ $\left.p_{2_{i}}\right|^{2}$. The algorithm we propose however is not dependent on this definition, and other distance metrics (such as $L_{\infty}$ ) can also be used.

Although the following results refer to data points in a 2-Dimensional space, they can easily be adapted for higher dimensions. In the following, we use the notation $R N N(q)$ to denote the answer to reverse nearest neighbor queries for point $q$. NN(q) returns the set of points that satisfy the nearest neighbor condition with respect to $q$. Note that both RNN(q) and NN(q) are sets of points in the database, while query point $q$ may or may not correspond to an actual data point in the data base.

Let query point $q=\left(x_{q}, y_{q}\right)$ be defined as having value $x_{q}$ on the x-coordinate and $y_{q}$ on the y -coordinate. Moreover, let three lines $l_{1}, l_{2}$, and $l_{3}$ intersect at $q$, and divide the space around it into six regions $S_{1}, S_{2}, \cdots S_{6}$
(Figure 2(a)). We fix $l_{1}$ to be parallel to the $x$ axis, and the angle between $l_{1}$ and $l_{2}, l_{2}$ and $l_{3}, l_{3}$ and $l_{1}$ to be exactly $60^{\circ}$. For the rest of the paper, we will refer to $l_{1}, l_{2}$ and $l_{3}$ as space dividing lines.
Proposition 1 : For a given 2-dimensional dataset, any query point $q, R N N(q)$ will return at most six data points [Smi97, KM99].

In higher dimensions the number of maximum data points that satisfy $R N N(q)$ is still a constant, whose value depends on the number of dimensions considered.

The result stated by Proposition 1 is based on the observation that the upper limit for the number of data points in $\operatorname{RNN}(q)$ is given by the scenario where all these points are on a circle with the center at $q$. The distance between two consecutive points on the circle must be the same as the radius of the circle, i.e., the distance between a point in $\mathrm{RNN}(\mathrm{q})$ and $q$.

(a) Dividing the space around query point $q$.

(b) Example for Proposition 3.

Figure 2:
A consequence of this observation is the following:
Proposition 2 : Let the space around a query point $q$ be divided into six equal regions $S_{i}(1 \leq i \leq 6)$ by
straight lines intersecting $q$. $S_{i}$ therefore is the space between two space dividing lines. Then:

## 1. There exist at most two RNN points in each region

 $S_{i}$2. if there exist exactly two $R N N$ points in a region $S_{i}$, then each point must be on one of the space dividing lines through $q$ delimiting $S_{i}$.
Proposition 3 : Let $p=N N(q)$ in $S_{i}$. If $p$ is not on a space dividing line, then either $N N(p)=q$ (and then $R N N(q)=p)$, or $\nexists R R N(q) \in S_{i}$.

Proof: To show that Prop. 3 holds, assume w.l.o.g. that there are two points in a region $S_{1}$ delimited by space dividing lines $l_{1}$ and $l_{2}$. Let point $p_{1}$ be an NN(q), $p_{1} \in S_{i}$ (Figure 2(b)). Let $p_{2}$ be another point in $S_{i}$ such that $p_{2}$ is RNN(q). We will show that either $\operatorname{dist}\left(q, p_{1}\right)=\operatorname{dist}\left(q, p_{2}\right)$ and $p_{1}$ on $l_{1}, p_{2}$ on $l_{2}$ (or viceversa), or $p_{1}$ and $p_{2}$ are the same data point. Assume $p_{1} \neq p_{2}$. Considering the triangle $\triangle q p_{1} p_{2}$, it must be the case that $p_{2}$ is closer to $q$ then to $p_{1}$. That is, $\operatorname{dist}\left(q, p_{2}\right) \leq \operatorname{dist}\left(p_{2}, p_{1}\right) \Longrightarrow\left(\widehat{p_{1} q p_{2}}\right) \leq\left(\widehat{q p_{1} p_{2}}\right) \leq 60^{\circ}$, and therefore angle $\left(\widetilde{p_{2} p_{1}}\right) \geq 60^{\circ}$. Since both $p_{1}$ and $p_{2}$ are in $S_{1}$, if the angle is $60^{\circ}$, it is the case that $p_{1}$ is on $l_{1}$ or $l_{2}$, while, by construction of the space dividing lines, $p_{2}$ must be on the opposite line delimiting $S_{i}\left(l_{2}\right.$ or $\left.l_{1}\right)$. Eliminating this case, we look now at the scenario where the angle between lines $q p_{1}$ and $q p_{2}$ is less than $60^{\circ}$ and $\left(q \widehat{p p}_{2} p_{1}\right)>60^{\circ}$. Because now $\left(q_{p_{2} p_{1}}\right)>\left(\widehat{q p_{1} p_{2}}\right)$ then it must be the case that $\operatorname{dist}\left(q p_{1}\right)>\operatorname{dist}\left(q p_{2}\right)$, which implies that $\mathrm{NN}(\mathrm{q})=p_{2}$ and $\neg\left(\mathrm{NN}(\mathrm{q})=p_{1}\right)$. However, this consequence contradicts the initial hypothesis that $\mathrm{NN}(\mathrm{q})=p_{1}$.
[]
The above proposition simply states that for a query point q and region $S_{i}$ either the nearest neighbor is also the reverse nearest neighbor, or there is no $\mathrm{RNN}(\mathrm{q})$ in $S_{i}$ (if $\mathrm{NN}(\mathrm{q})$ is not on the space dividing lines). This is a very important result, since it allows us to have a criterion for limiting the choice of RNN(q) to one or two points in each of the six regions $S_{i}$. An algorithm for answering an RNN(q) query then first finds the candidate set of data points: one or two points in each region $S_{i}$ that satisfy the nearest neighbor condition $\mathrm{NN}(\mathrm{q})$. In the second step, these candidate points are tested for the condition that their nearest neighbor is $q$. The points that satisfy this condition are returned as the answer to the $\mathrm{RNN}(\mathrm{q})$ query.

### 3.2 Algorithm Development

In the past years several papers developed index structures based on R-trees, and discussed possible optimizations ([SR87, Gut84, BKSS90, KF93, KF94, BKK96, PF99, EKK00]. [RKV95] proposed a branch-and-bound method to find the nearest neighbor to a


Figure 3: A section of an R-tree and the corresponding regions $S_{i}$ and minimum bounding rectangles
query point, based on the traversal and pruning of the R-tree indexing structure. The intuition behind it is that points in a data space can be grouped in clusters delimited by minimum bounding rectangles (MBR), which in turn can be part of larger clusters. An MBR is the smallest rectangle completely enclosing a set of points. Every face on the MBR then contains at least one data point. Since an MBR is rectangular in shape, it can be described by the coordinates of two of its opposite corners. A node in the R-tree then stores these coordinates, and the levels of the R-tree reflect the granularity of the clusters considered (Figure 3). An algorithm to find NN(q) of a query point $q$ searches down the R -tree, by comparing the nodes at each level. Only the nodes that can lead to nearest neighbor points are further considered, while the rest of the tree is ignored. In [RKV95] the proximity comparison is based on " mindist", the shortest distance from the query point to the rectangular cluster as well as the "minmaxdist" (the minimum over all dimensions, of the distance from the query point to the furthest point on the closest face of the MBR). "Mindist" guarantees that all points in the MBR have at least "mindist" distance from the query point. "Minmaxdist" ensures that there exists at least one data point in the MBR whose distance to the query point is at most "minmaxdist". More details are in Section 3.3. Also, [RKV95] proposes to use the square of the Euclidean distance as a distance measure, because of the reduced computation cost.

Our approach for efficiently answering RNN queries builds upon the previous work on NN queries. Since we believe that NN queries are frequent and related to

RNN queries, we can safely assume that a method for answering NN queries is already implemented in most systems. The solution described in this paper follows the algorithm described in [RKV95], but it can be easily modified to make use of other techniques developed for NN queries.

The method we propose finds for each region $S_{i}$, $1 \leq i \leq 6$, (see Section 3.1) the points $p=N N(q)$ that satisfy the condition $p \in S_{i}$. The set of data points $p=N N(q)$ is the result of the algorithm for nearest neighbor search applied to the data set, but considering only the projections on the desired dimensions. Due to Prop. 3 (see Section 3.1), in a 2-Dimensional space only one or two such points can satisfy the reverse nearest neighbor condition. We modify the algorithm used to find nearest neighbor points by introducing new restrictions during the pruning of the indexing tree. While traversing the R-tree, a node is first checked for inclusion in $S_{i}$, then it is checked for proximity to query point $q$. The subtrees rooted at nodes that do not satisfy both of these conditions are then pruned from the node set of interest. Following the traversal of the R-tree for each region $S_{i}$ in the data space, a set of candidate RNN(q) points are returned. Not all candidate points $p$ are indeed reverse nearest neighbors of $q$, i.e., satisfy the condition that $q=\mathrm{NN}(\mathrm{p})$. Since the query point $q$ is not necessarily one of the existing data points, we test that $q=\mathrm{NN}(\mathrm{p})$ by comparing the distance $\operatorname{dist}(q, p)$ with the distance between p and its nearest neighbor $\operatorname{dist}(p, N N(p))$. It is important to note that, if a data point falls on one of the space dividing lines, then it belongs to two of the $S_{i}$ 's. The same points $p=R N N(q)$ can therefore be returned by different iterations of the tree traversal, and a filtering step is needed to exclude such duplicates.

To summarize the above modifications to an algorithm designed for NN queries, the following is an overview. Let a query point be $q=\left(x_{q}, y_{q}\right)$

1. Construct the space dividing lines $l_{1}, l_{2}$ and $l_{3}$ such that:
(a) $l_{1}, l_{2}$ and $l_{3}$ intersect at the query point $\left(x_{q}, y_{q}\right)$
(b) angle $\left(l_{1}, l_{2}\right)=\operatorname{angle}\left(l_{2}, l_{3}\right)=\operatorname{angle}\left(l_{3}, l_{1}\right)=60^{\circ}$
2. For each $S_{i}$ (see Section 3.1) do
(a) traverse R-tree using the conditional NN algorithm (Section 3.3. That is, find $\mathrm{p}=\mathrm{NN}(\mathrm{q})$ such that $p \in S_{i}$
(b) if $\mathrm{NN}(\mathrm{p})=\mathrm{p}^{\prime}$ where $\operatorname{dist}\left(p p^{\prime}\right) \geq \operatorname{dist}(p q)$, add $p$ to the list of RNN(q)
3. Eliminate duplicates in $\mathrm{RNN}(\mathrm{q})$

Step $2(a)$ is not straight-forward, since it entails conditions ( $p \in S_{i}$ ) on the algorithm that finds $\mathrm{p}=$ $\mathrm{NN}(\mathrm{q})$. In the next section we present the modifications imposed on the NN algorithm in order to support conditional NN searches that find the nearest neighbor data point restricted to a region of the space.


Figure 4:

### 3.3 Conditional NN Queries

In this section we analyze nearest neighbor queries constrained to a certain region. Although it is generally an important problem to solve for different types of constraints, for the purpose of the RNN algorithm we consider only linear constraints. We are interested in the special case of finding a nearest neighbor in each of the six regions divided by space dividing lines through a query point. A brute-force algorithm that finds all the nearest neighbors until there is one in the queried region $S$, can be very inefficient in scenarios like the one described in Figure 4(a). In this case, since data point $p_{2}$ in $S$ is actually the furthest from the query point $q$, a brute-force algorithm would first consider and then discard all other points before finding $p_{2}$.

A cluster or an equivalent $M B R$ in the context of Rtrees, belongs to a region either fully or partially. In Figure 4(b) we show the five different cases where an MBR (minimum bounding rectangle) is considered to
overlap with a region $S_{2}$ :

1. MBR A: fully contained in $S_{2}$
2. MBR B: three vertices are in $S_{2}$. Since $B$ is a minimum bounding rectangle, then there must be some data points in $B$ contained also in $S_{2}$
3. MBR C: two of $C$ 's vertices are in $S_{2}$. This means that an entire edge $e$ of the MBR is in $S_{2}$, which by definition guarantees that at least one data point in $C$ is on $e$ and therefore in $S_{2}$.
4. MBR D: only one vertex of $D$ is in $S_{2}$. It is important to note that in this case there is no implication on the existence of data points contained both in $D$ and in $S_{2}$.
5. MBR E: although no vertices are in $S_{2}$, part of the $E$ minimum bounding rectangle overlaps with the queried region $S$. Again, as in the previous case, we cannot make any assumption that any of the data points contained in $E$ are also contained in $S$.

The approach we use to find a nearest neighbor in a region $S_{i}$ is to traverse the indexing tree and prune out sections of the tree that cannot lead to an answer either because the MBRs do not belong to the queried region, or because there is a guarantee that other points in $S_{i}$ are closer to the query point.

If the space constraints are eliminated, the problem of conditional nearest neighbor search is extended from a region to the whole space. The traditional NN methods are therefore only a special case of a more general approach. We will show however that, with only minor modifications, we can extend an NN search algorithm [RKV95] to satisfy conditional nearest neighbor queries. The method described in [RKV95] traverses recursively the indexing tree in depth first order. At each nonleaf level it creates a list of the node's children and sorts it according to the distance metric used. For efficiency, the subtrees rooted at nodes in the list will be visited in the sorted order. Once the recursive function on one of the children returns, a candidate for the nearest neighbor is returned and the list of remaining children to be visited is pruned accordingly. The difference between the algorithm proposed by [RKV95] and a conditional nearest neighbor algorithm resides only in the metric used to sort and prune the list of candidate nodes. The idea behind the former method is that if we define the mindist $(q, M)$ to be the minimum distance from the query point to an MBR $M$ and minmaxdist $(q, M)$ to be the minimum over all dimensions, of the distance from $q$ to the furthest vertex on the closest plane of $M$, then it is guaranteed that there exists at least a data point within the $[\operatorname{mindist}(\mathrm{q}, \mathrm{M}), \operatorname{minmaxdist}(\mathrm{q}, \mathrm{M})]$ range. Then another $\operatorname{MBR} M^{\prime}$ whose $\operatorname{mindist}\left(q, M^{\prime}\right)>\operatorname{minmaxdist}(q, M)$,


Figure 5: minmaxdist $(q, M)$ is modified to minmaxdist $(q, M, S)$ for conditional NN queries
can be safely discarded. Following the same reasoning, a data point $p$ whose distance to the query point $\operatorname{dist}(q, p)>\operatorname{minmaxdist}(q, M)$, can also be discarded from the list of candidates to the nearest neighbor. Finally, if there is a point $p$ that satisfies the condition that $\operatorname{dist}(q, p)<\operatorname{mindist}(q, M)$, then the minimum bounding box $M$ is discarded.

The method described above assumes that the entire volume of a minimum bounding rectangle is contained in the queried region. This is not the case for conditional NN queries, and a few modifications are necessary. In the table below we described five possible positions of MBRs with respect to a region $S_{2}$, cases that should be considered when trying to find NN(q) in $S_{2}$. Our goal is to modify the notion of minmaxdist $(q, M)$ to have the same implications and therefore be used in the same manner as proposed by [RKV95]. Note that the measure minmaxdist $(q, M)$ may be irrelevant if the MBR $M$ is only partially contained by the the queried region. Since we do accommodate these cases, we redefine $\operatorname{minmaxdist}(q, M)$ to be $\operatorname{minmaxdist}(q, M, S)$, the distance from the query point to the furthest vertex of $M$ on its closest face included in $S$.

If an MBR $M$ that intersects region $S$ has one or no vertices in $S$, then there is no guarantee that
there are data points in both $M$ and $S$. We then set minmaxdist $=\infty$ to capture this uncertainty. Another case is if $M$ has two or three vertices in $S$, which implies that at least an edge and therefore at least one data point is in both $M$ and $S$. This edge is not necessarily the closest to the query point $q$, and consequently we have to redefine minmaxdist as a function of the distance from $q$ to the furthest vertex of the closest face in region $S$.

The computation of mindist $(q, M)$ is valid for all cases, since it provides a definite lower bound on the location of data points inside an MBR. For cases where this distance is measured to a point on the MBR that falls outside of region $S$, the mindist still gives a correct although looser lower bound.

The use of a generic R-tree permits us to easily ignore certain dimensions if the RNN query is not interested in them. In particular, mindist() and minmaxdist() can be defined to simply consider distance to the relevant dimensions only. Hence, depending on the query, alternative dimensions can be considered. This is much more flexible then an approach where a tree is constructed based on the dimensions of interest determined a-priori.

### 3.4 Algorithm

The RNN algorithm developed in this paper starts by finding one or two nearest neighbors in each of the six regions $S_{i}$, and then it tests if these data points indeed satisfy the RNN condition. For clarity, we described how to find the candidate points separately for each region. In reality, the computation corresponding to all the six regions can be done in one traversal of the indexing tree. Separate priority lists should be created for each region, but the information about the minimum bounding rectangles will only be read once. Similarly, the RNN candidates can be verified in one traversal of the indexing tree. We next present the algorithm for finding RNN points.
RNNsearch(). This is the main function that first creates the structure that holds the information on the query point in the necessary format, and initializes nearest, the nearest neighbor list. It then calls CondNNSearch which returns a list of nearest neighbors for each of the regions. The duplicates in this list are eliminated (EliminateDuplicates()), and the NNSearch function is used to check if indeed these candidates are RNN points. That is, we test if the query point is a nearest neighbor of the points returned by CondNNSearch. For clarity we only describe how the conditional nearest neighbor search is done in parallel. However, in practice we can use CondNNSearch for both conditional and unconditional nearest neighbor searches, and use a
flag condFlag to distinguish between the two. In the algorithm below we do not integrate the two functions and we do not show the details for running $N N$ Search in parallel, but the method is very similar to that of parallelizing CondNNSearch. The number of regions, numSections, depends on the number of dimensions of the queried space, which is inferred from the format of the query point passed as an argument to the function. Since this information is used to define the dimensions of the query space, the dimensions that are not necessary are ignored during the computation of the distance between two MBRs.

## Search for Conditional Nearest Neighbor

(CondNNSearch). This is a recursive method for searching the conditional nearest neighbors of a query point, and it is based on [RKV95]. When the function CondNNSearch is passed a pointer to a node in the indexing tree, it builds a set of lists branchList (branchList[0] ... branchList[numSections1]) for each region that is searched. The branch lists have pointers to the children of node. An additional list, branchList[numSections] of pointers to the children of node contains counters of pointers from the other branch lists for each of the children. Children with a higher count are visited first, since this step is required by the search of most regions and the I/O overhead can thus be reduced. The branch lists are first pruned (CondDownPrune()) based on the nearest neighbor found so far and also based on the relationship between the MBRs contained by the respective children. In this step, according to the distance metric used, the MBRs that cannot contain nearest neighbors are excluded. CondNNSearch is recursively called on the child with a highest count, i.e., that most of the branch lists point to. The function will return an updated list of nearest neighbors, with more than one element if the respective data points are at the same distance from the query point. The new nearest neighbor candidate will be used again to prune the branch lists and refresh the count list (CondUpPrune()). Both branch pruning functions CondDownPrune and CondUpPrune use the distance comparison criterion discussed in Section 3.3. If the branch lists are not null, the next child in decreasing order of the count in branchList[numSetions] will be visited. If node is a leaf of the index tree, then CondNNSearch only compares the distance from the children to the nearest points and, if necessary, updates the structure nearest that stores the list of nearest neighbors.

## 4 Conclusion

In the approach proposed by [KM99], an additional tree is constructed to store a set of objects consisting of the data points and their distance to the corresponding nearest neighbors, and an RNN(q) query is

| Number of vertices in $S_{i}$ | $\operatorname{mindist}\left(q, M, S_{i}\right)$ | $\operatorname{minmaxdist}\left(q, M, S_{i}\right)$ |
| :--- | :--- | :--- |
| 0 (no intersection of M with $\left.S_{i}\right)$ | $\infty$ | $\infty$ |
| 0 (M intersects $\left.S_{i}\right)$ | $\operatorname{mindist}(q, M)$ | $\infty$ |
| 1 |  |  |
| 2 | $\operatorname{mindist}(q, M)$ | minmaxdist $\left(q, M, S_{i}\right)=$ distance to |
| 3 |  | furthest vertex on closest face IN $S_{i}$ |
| 4 |  |  |

Table 1: Definition of $\operatorname{mindidst}\left(q, M, S_{i}\right)$ and $\operatorname{minmaxdist}\left(q, M, S_{i}\right)$

```
/* --------------------------------------------------------------------------------
|RNNsearch: returns a list of reverse nearest neighbors of a quey point $q$
struct Nearest RNNsearch(q)
struct Rectangle *q;
{
    struct Nearest *nearest, *newnearest, *RNNresult;
    int newRNN; /*number of RNN candidates*/
    Initialize(nearest);
    /*return a list of RNN candidates*/
    NNCondSearch(TreeRoot,q, nearest, numSections, NODECARD,0);
    /*eliminate duplicates from the RNN list and return the count of remaining candidates*/
    numRNN = EliminateDuplicates(nearest);
    for i=0 to numRNN do
        {
        NNSearch(TreeRoot,nearest[i],newnearest);
        /*if the candidate nearest[i] is indeed an RNN, add it to the list*/
        if Rect2Rectdistance(q, &newnearest[i]->branch[0].rect) == Rect2Rectdistance(q,nearest->branch[0].rect)
            Append(RNNresult, nearest[i]);
        }
    return RNNresult;
}
```

Table 2: Algorithm for Reverse Nearest Neighbor search

```
/* --------------------------------------------------------------------------------------------
|CondNNSearch: recursive conditional nearest neighbor search. Based
|on the query point ("point") and the number of dimensions, divide
|the space into a number numSections of regions and search in parallel
|for the nearest neighbors.
---------------------------------------------------------------------*/
CondNNSearch(node, point,nearest,numSections,NODECARD, condFlag)
Node *node;
Rectangle *point;
struct Nearest nearest;
int numSections,NODECARD; /*NODECARD is the number of a node's children*/
boolean condFlag; /*distinguishes between conditional and unconditional search. We
    do not explicitly use it in this presentation*/
{
    branch[numSections+1][NODECARD] branchList;
    int i,j,k, dist, nearestDist;
    if (node->level >= 0) /*this is an internal node*/
    {
            /*generate the six branchLists*/
            for i:=0 to NODECARD do
            for k:=O to numSections do
                Update(branchList[k]);
            /*perform downward pruning*/
            CondDownPrune(point, nearest, branchList);
            /*create branchList[numSections], the ordered counter list*/
            UpdateCount(branchList);
            for j:=0 to NODECARD do
            {
                newnode = branchList[numSections][0]->child;
            /*recursively visit children*/
            CondNNSearch(newnode,point, nearest,numSections,NODECARD,condFlag);
            /*perform upward pruning of the branch lists*/
            CondUpPrune(point, nearest, branchList);
            /*update the ordered counter list, branchList[numSections]*/
            UpdateCount(branchList);
            }
    }
    else /*this is a leaf node*/
            for i:=O to NODECARD
                    if (node->branch[i].child != NULL)
            {
                    /*"dist" is the distance between node->branch[i].rect and query point*/
                    dist = Rect2Rectdistance(point, &node->branch[i].rect);
                    nearestDist = Rect2Rectdistance(point, &nearest[0]->branch[0].rect)
                    /*if nearest has not been set yet or it represents a rectangle
                    further from the query point than node->branch[i]*/
                    if ((nearest[0].rect == NULL)||(dist < nearestDist))
                    /*if branch[i].rect is a possible nearest neighbor, add it to the NN list*/
                    else if (dist == Rect2Rectdistance(point, &nearest->branch[i].rect))
                    Append(nearest, branch->[i].rect);
            }
}
```

Table 3: Algorithm for Conditional Nearest Neighbor search
answered by testing the enclosure of $q$ within these objects. Therefore, only one traversal of the RNN tree is necessary to compute answers to RNN queries. However, the drawbacks to this approach arise if updates are allowed to modify the database. To maintain the advantage of fast query answers, sections of both NN and RNN trees should be modified with every insert and delete operation. We choose to develop an algorithmic method, since we are specifically interested in dynamic databases, where updates are frequent and queries have the flexibility to refer only to a subset of the data space dimensions. Our method does not require the construction and maintenance of any additional data structures and only the underlying indexing tree is updated. We believe that in dynamic databases the disadvantage of a higher cost during query computation is outweighed by the simplicity and efficiency of updates. In our future plans we include a performance comparison with the original solution [KM99]. Also, we are interested in exploring alternative implementations based on R-trees, to concurrently evaluate a wider range of queries.

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