

Κινητός και Διάχυτος Υπολογισμός (Mobile & Pervasive Computing)

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Διάλεξη 11η

Ιστοσελίδα του μαθήματος

- http://skyblue.csd.auth.gr/~dimitris/courses/mpc_fall05.htm
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- Τοποθετούνται οι διαφάνειες του επόμενου μαθήματος
- Τοποθετούνται τα research papers που αντιστοιχούν σε κάθε διάλεξη. Τα σημαντικά με πρόθεμα **MUST_BE_READ**

Περιεχόμενα

- Ασύρματα Δίκτυα Αισθητήρων
(Wireless Sensor Networks)
 - Πρόβλημα κάλυψης και
συνδεσμικότητας σε ασύρματα δίκτυα
αισθητήρων

Coverage Problems

- In general
 - Determine how well the sensing field is monitored or tracked by sensors.
- Possible Approaches
 - Geometric Problems
 - Level of Exposure
 - Area Coverage
 - Coverage
 - Coverage and Connectivity
 - Coverage-Preserving and Energy-Conserving Problem

Review: Art Gallery Problem

- Place the minimum number of cameras such that every point in the art gallery is monitored by at least one camera.

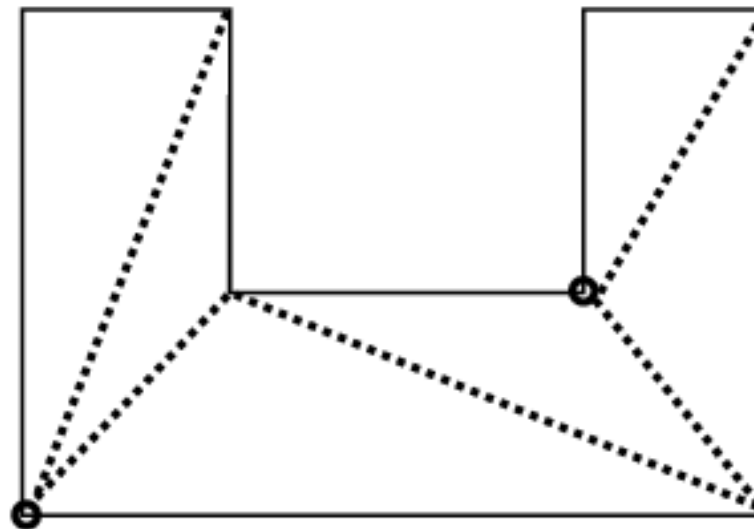


Figure 1: An example of triangulating a polygon and a possible deployment of cameras. Circles represent positions of cameras.

Review: Circle Covering Problem

- Given a fixed number of identical circles, the goal is to minimize the radius of circles.

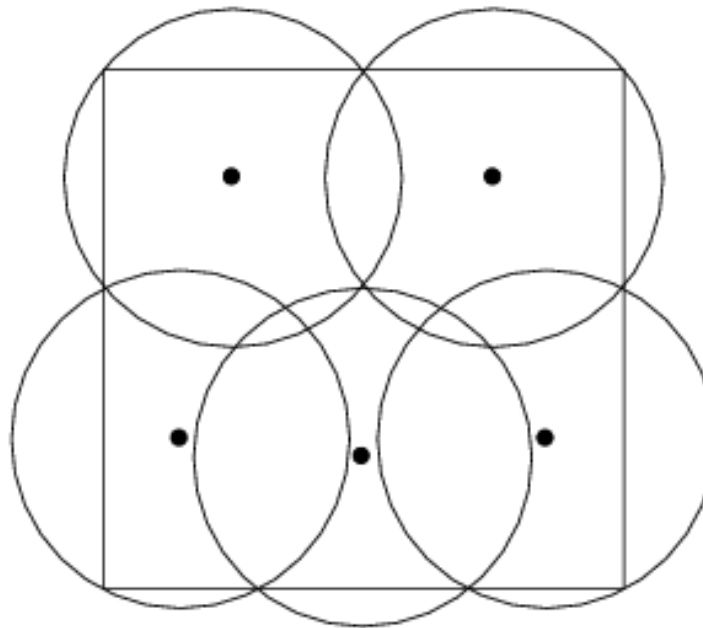
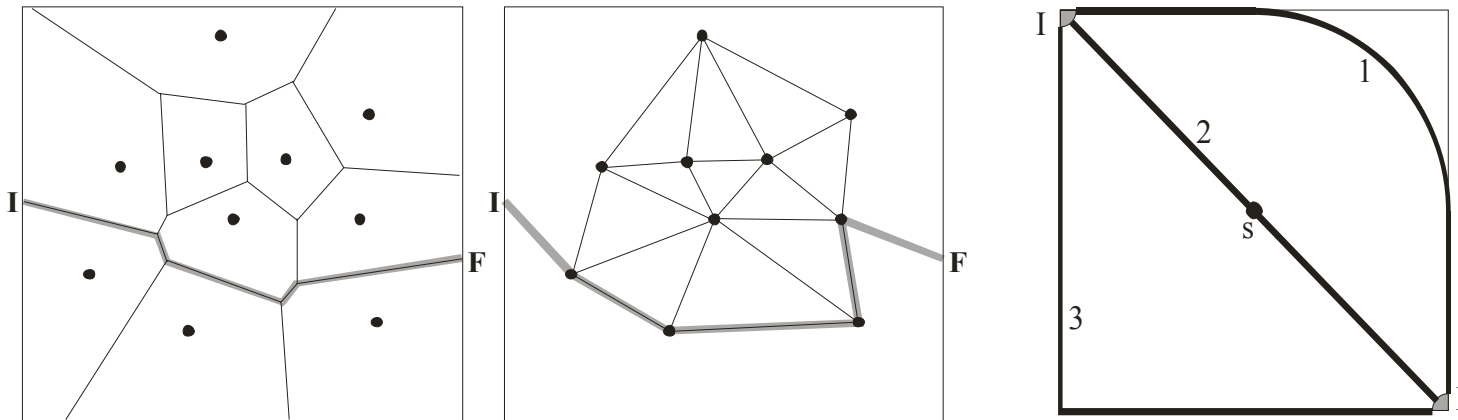


Figure 2: An example of an optimal covering with 5 circles. The radius of each circle is about 0.3621605.

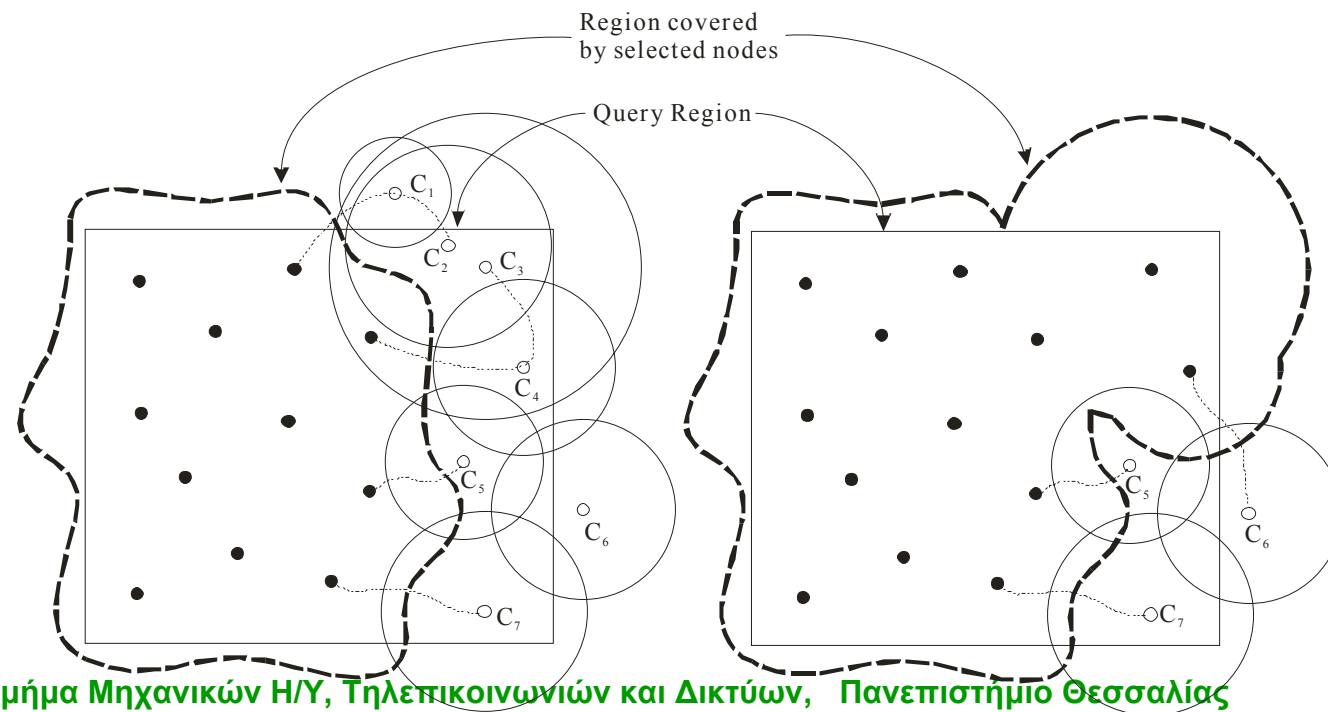
Level of Exposure

- Breach and support paths
 - paths on which the distance from any point to the closest sensor is maximized and minimized
 - Voronoi diagram and Delaunay triangulation
- Exposure paths
 - Consider the duration that an object is monitored by sensors



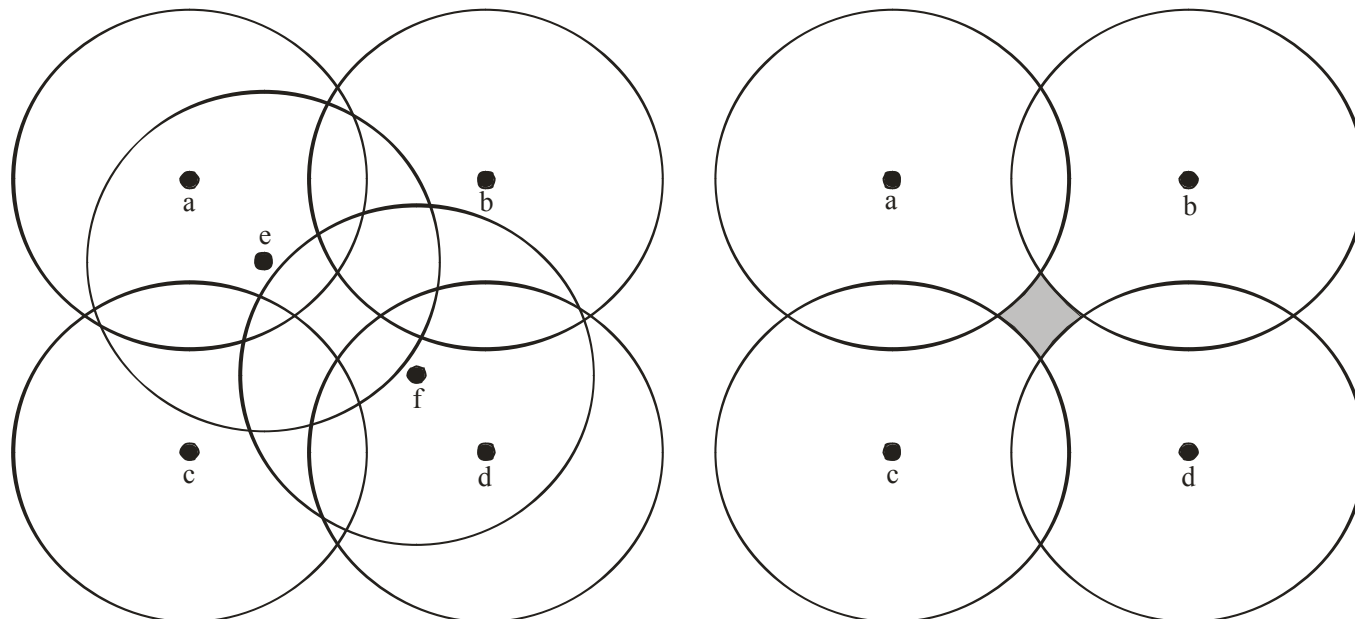
Coverage and Connectivity

- A region is k -covered, then the sensor network is k -connected if $R_C \geq 2R_S$
- Extending the coverage such that connectivity is maintained.



Coverage-Preserving and Energy-Conserving Protocols

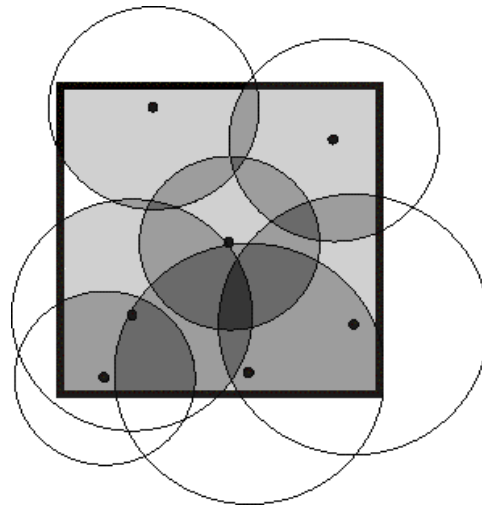
- Sensors' on-duty time should be properly scheduled to conserve energy.
 - thus extending the lifetime of the network.
 - This can be done if some nodes share the common sensing region.



The Coverage Problems in 2D Spaces

Coverage Problems

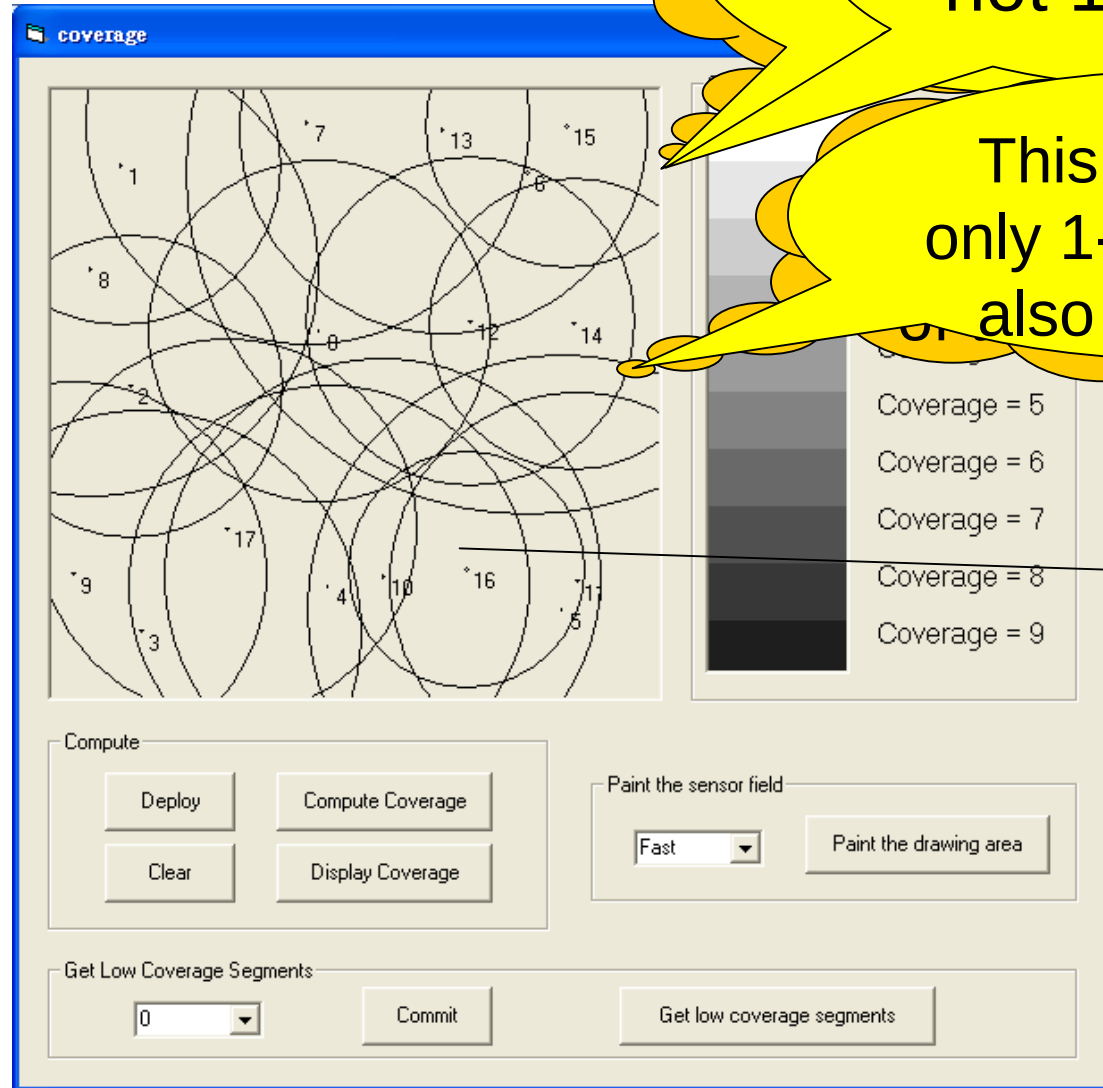
- In general
 - To determine how well the sensing field is monitored or tracked by sensors
 - Sensors may be randomly deployed



Coverage Problems

- We formulate this problem as
 - Determine whether every point in the service area of the sensor network is covered by at least α sensors
 - Why α sensors?
 - Localization, positioning, and tracking applications
 - Fault-tolerance

The 2D Cov



So this area is not 1-covered!

This area is not only 1-covered, but also 2-covered!

me 2-covered that means that every point in this area is covered by at least 2 sensors

Coverage level = α means that this area is α -covered

Sensing and Communication Ranges

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Radio transmission range of Berkeley Motes

Product	Transmission Range
MPR300*	30m
MPR400CB	150m
MPR410CB	300m
MPR420CB	300m
MPR500CA	150m
MPR510CA	300m
MPR520CA	300m

Table 2: Sensing range of several typical sensors

Product	Sensing Range	Functions
HMC1002 Magnetometer sensor [8]	5m	Detecting disturbance from Automobiles
Reflective type photoelectric sensor [2]	1m	Detecting targets of virtually any material
Thru-beam type photoelectric sensor [2]	10m	Detecting targets of virtually any material
Pyroelectric infrared sensor (RE814S) [18]	30m	Detecting moving objects
Acoustic sensor Berkeley Motes * [8]	~ 1m	Detecting acoustic on sound sources

* This result is based on our own measurement on Berkeley motes [8].

¹Honghai Zhang and Jennifer C. Hou, "On deriving the upper bound of α -lifetime for large sensor networks," *Proc. ACM Mobihoc 2004*, June 2004

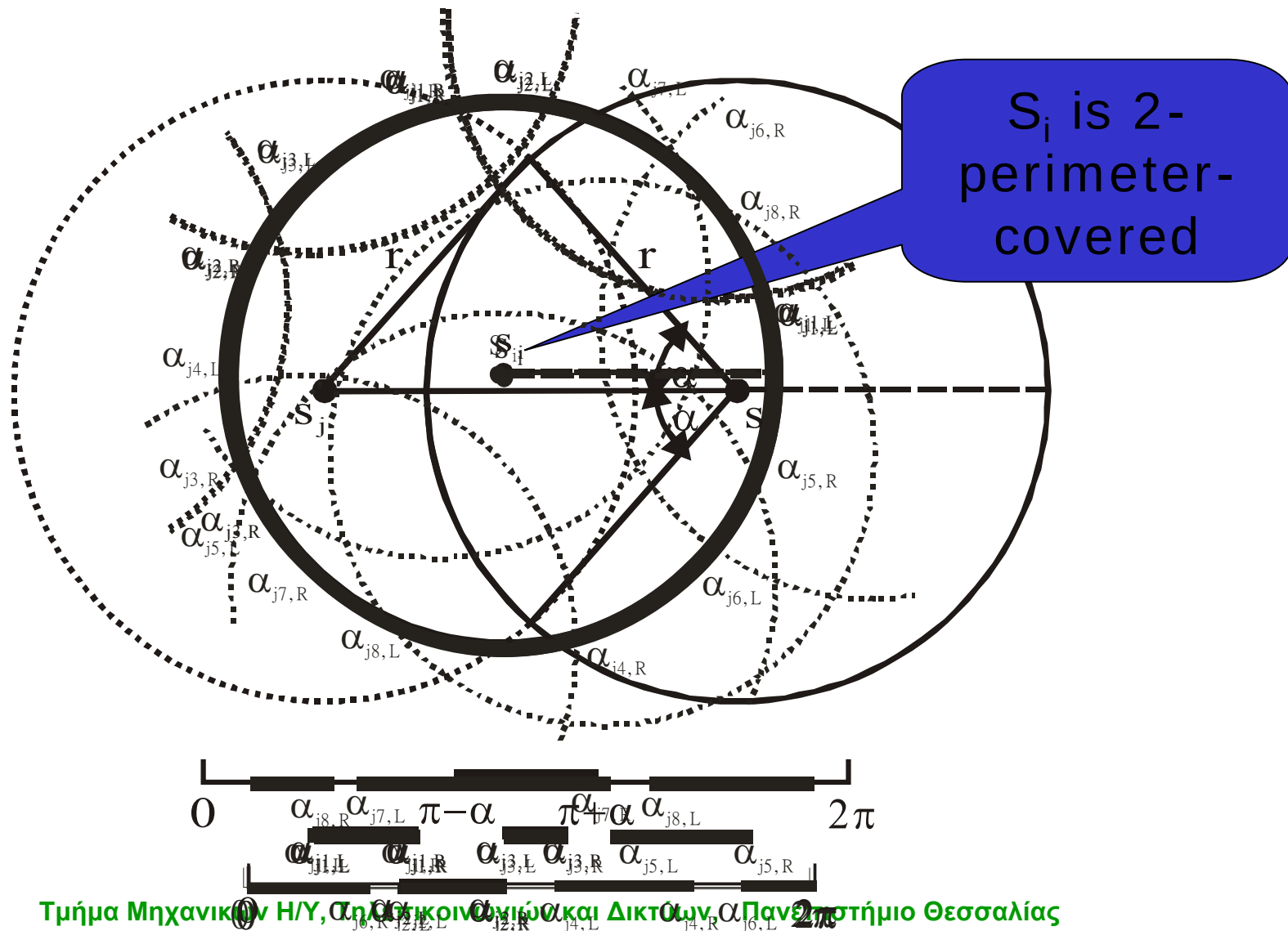
Assumptions

- Each sensor is aware of its geographic location and sensing radius.
- Each sensor can communicate with its neighbors.
- Difficulties:
 - $O(n^2)$ regions divided by n circles
 - How to determine boundaries of these regions?

The Proposed Solution

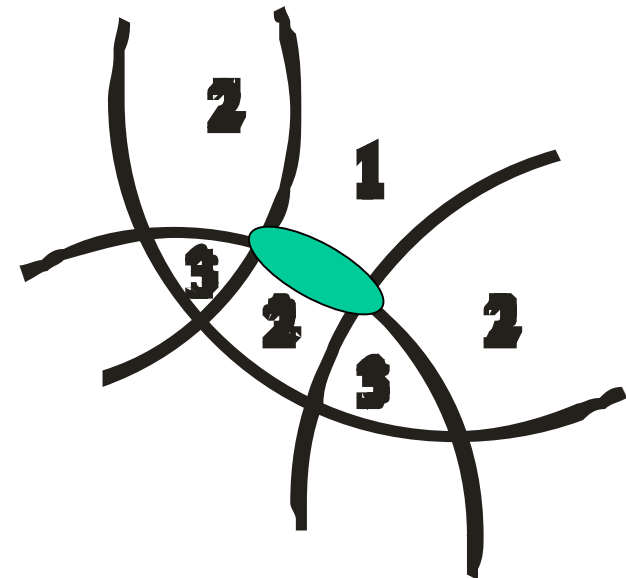
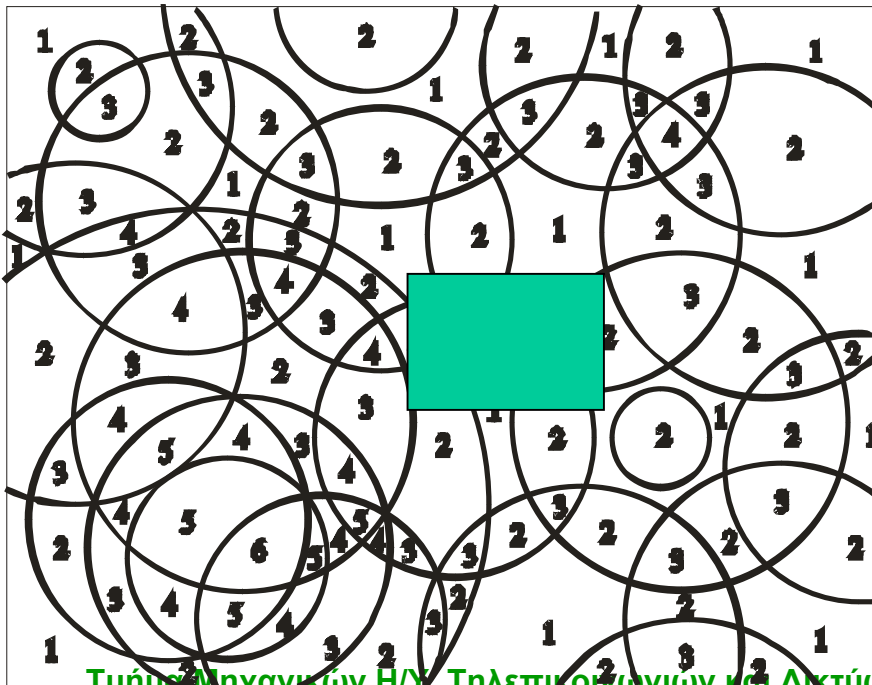
- We try to look at how the **perimeter** of each sensor's sensing range is covered.
 - How a perimeter is covered implies how an area is covered
 - ... by the continuity of coverage of a region
- By collecting **perimeter coverage** of each sensor, the level of **coverage of an area** can be determined.
 - a distributed solution

How to calculate the perimeter cover of a sensor?



Relationship between k -covered and k -perimeter-covered

- THEOREM. Suppose that no two sensors are located in the same location. The whole network area A is **k -covered** iff each sensor in the network is **k -perimeter-covered**.



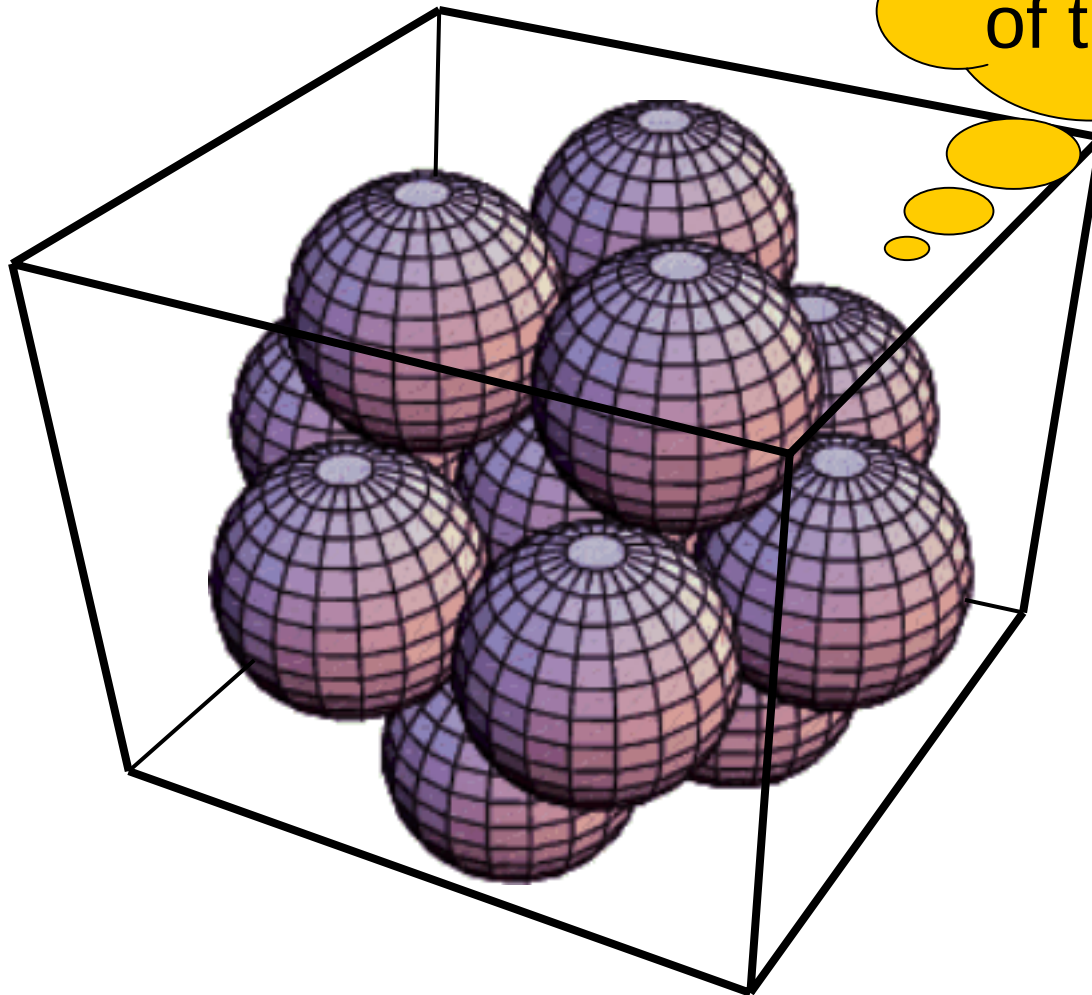
Detailed Algorithm

- Each sensor independently calculates its perimeter-covered.
 - $k = \min \{\text{each sensor's perimeter coverage}\}$
- Time complexity: $nd \log(d)$
 - n : number of sensors
 - d : number of neighbors of a sensor

The Coverage Problem in 3D Spaces

The 3D Coverage

What is the coverage level of this 3D area?



The 3D Coverage Problem

- Problem Definition
 - Given a set of sensors in a 3D sensing field, is every point in this field covered by at least α sensors?
- Assumptions:
 - Each sensor is aware of its own location as well as its neighbors' locations.
 - The sensing range of each sensor is modeled by a **3D ball**.
 - The sensing ranges of sensors can be non-uniform.

Overview of Our Solution

- The Proposed Solution
 - We reduce the geometric problem from a **3D space** to one in a **2D space**, and further to one in a **1D space**.

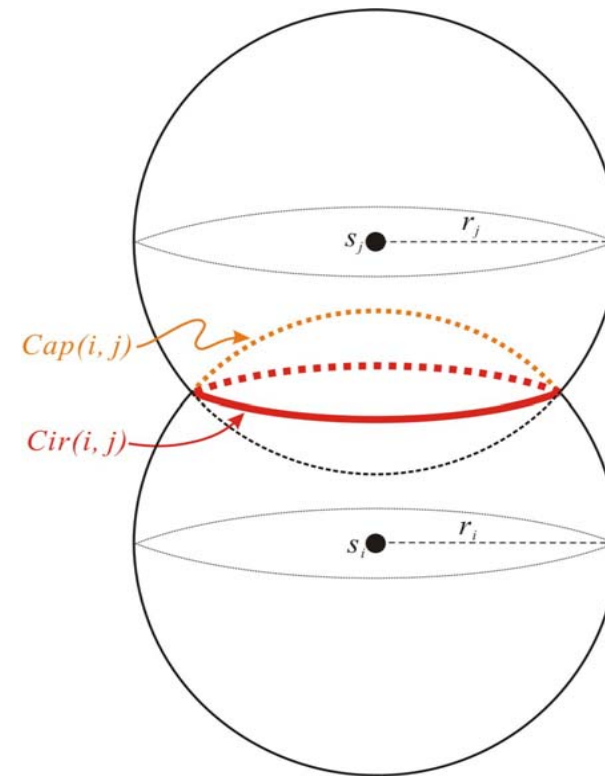
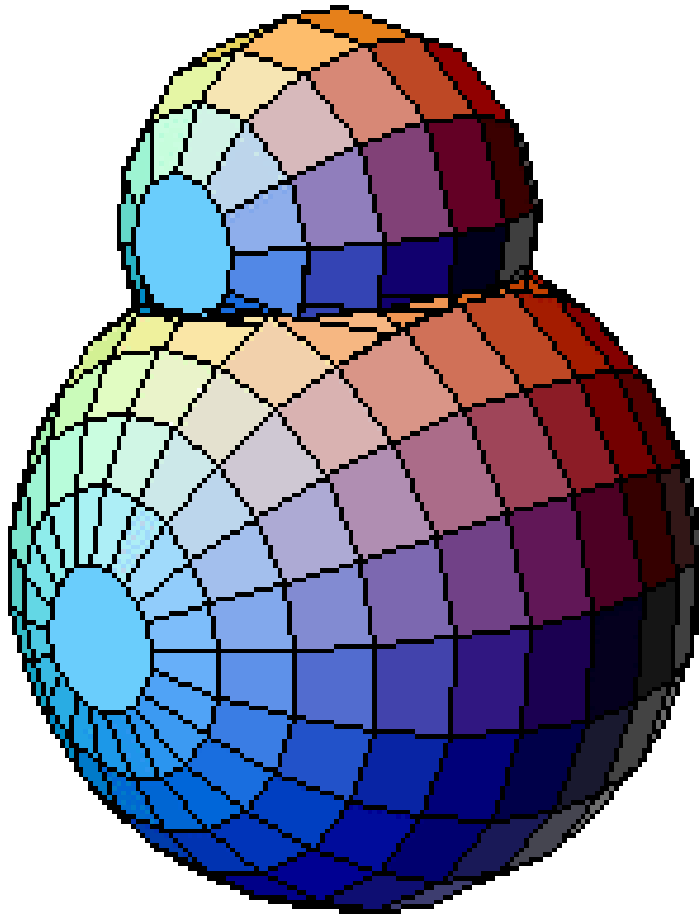
Reduction Technique (I)

- $3D \Rightarrow 2D$
 - To determine whether the whole sensing field is sufficiently covered, we look at the **spheres** of all sensors
 - Theorem 1: If each sphere is **α -sphere-covered**, then the sensing field is **α -covered**.
 - Sensor s_i is **α -sphere-covered** if all points on its sphere are **sphere-covered** by at least α sensors, i.e., on or within the spheres of at least α sensors.

Reduction Technique (II)

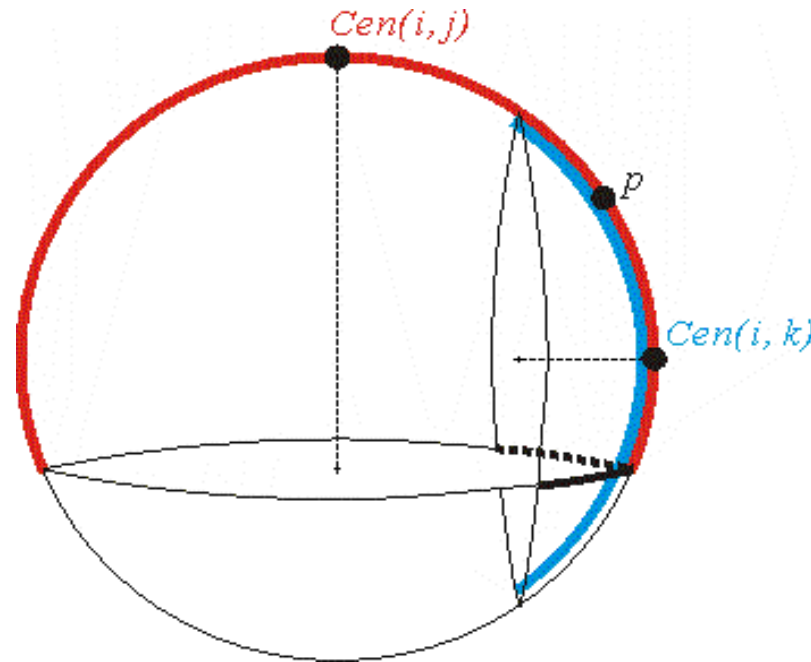
- 2D $\Rightarrow$ 1D
 - To determine whether each sensor's sphere is sufficiently covered, we look at how each **spherical cap** and how each **circle** of the intersection of two spheres is covered.
 - (refer to the next page)
 - Corollary 1: Consider any sensor s_i . If each point on S_i is **α -cap-covered**, then sphere S_i is **α -sphere-covered**.
 - A point p is **α -cap-covered** if it is on at least α caps.

Cap and Circle



k -cap-covered

- p is *2-cap-covered* (by $Cap(i, j)$ and $Cap(i, k)$).

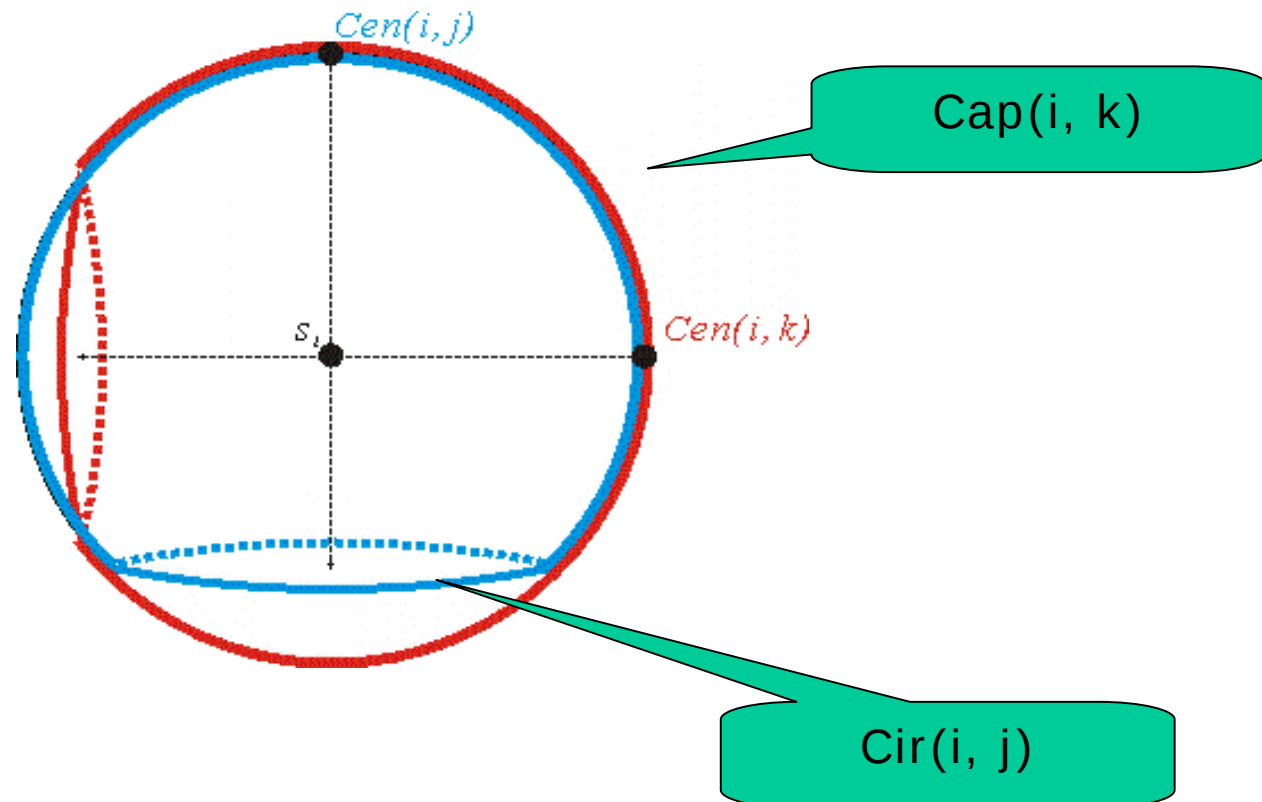


Reduction Technique (III)

- $2D \Rightarrow 1D$
 - Theorem 2: Consider any sensor s_i and each of its neighboring sensor s_j . If each circle $Cir(i, j)$ is *α -circle-covered*, then the sphere S_i is *α -cap-covered*.
 - A circle is *α -circle-covered* if every point on this circle is covered by at least α caps.

k-circle-covered

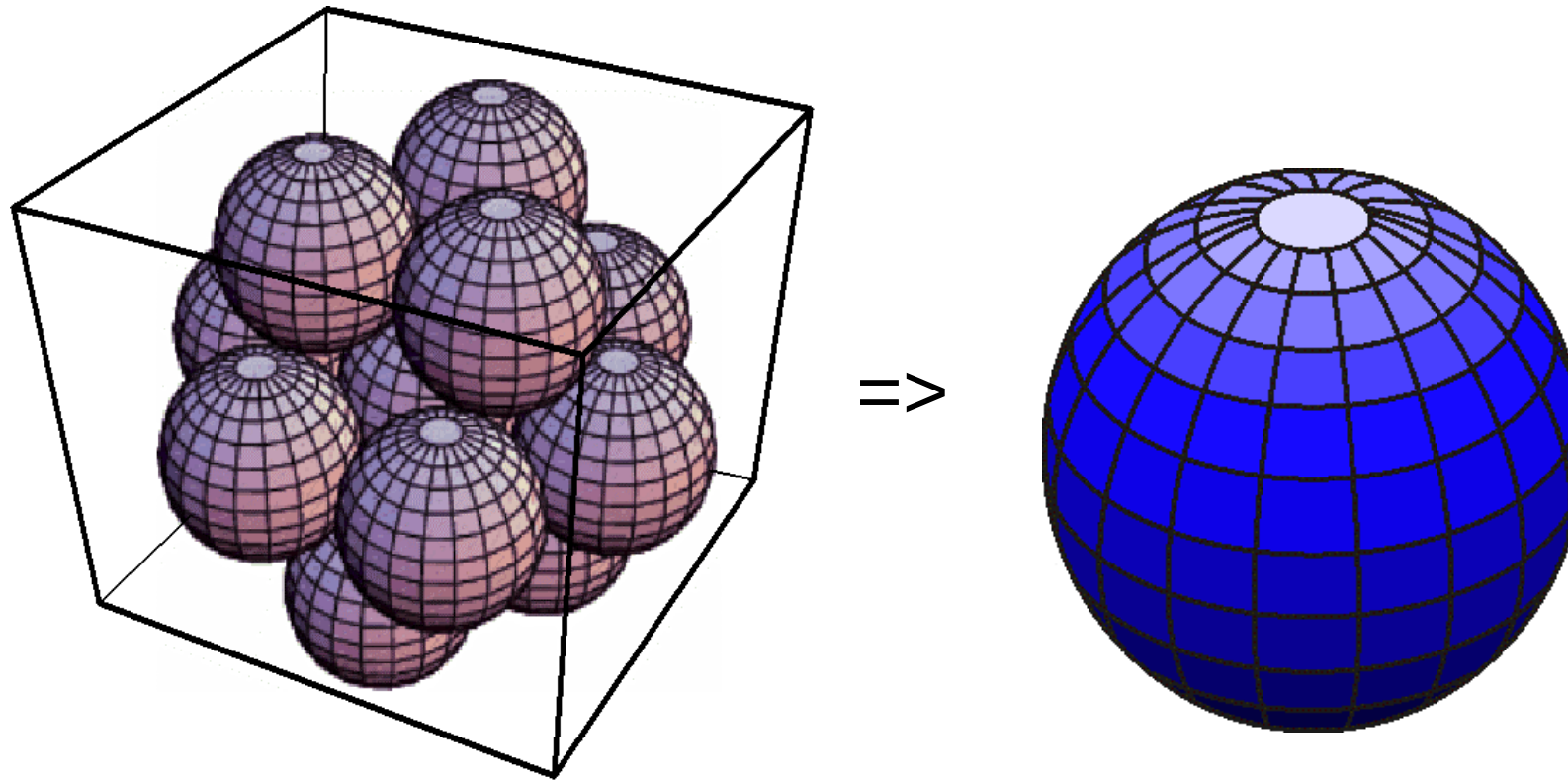
- $Cir(i, j)$ is *1-circle-covered* (by $Cap(i, k)$).



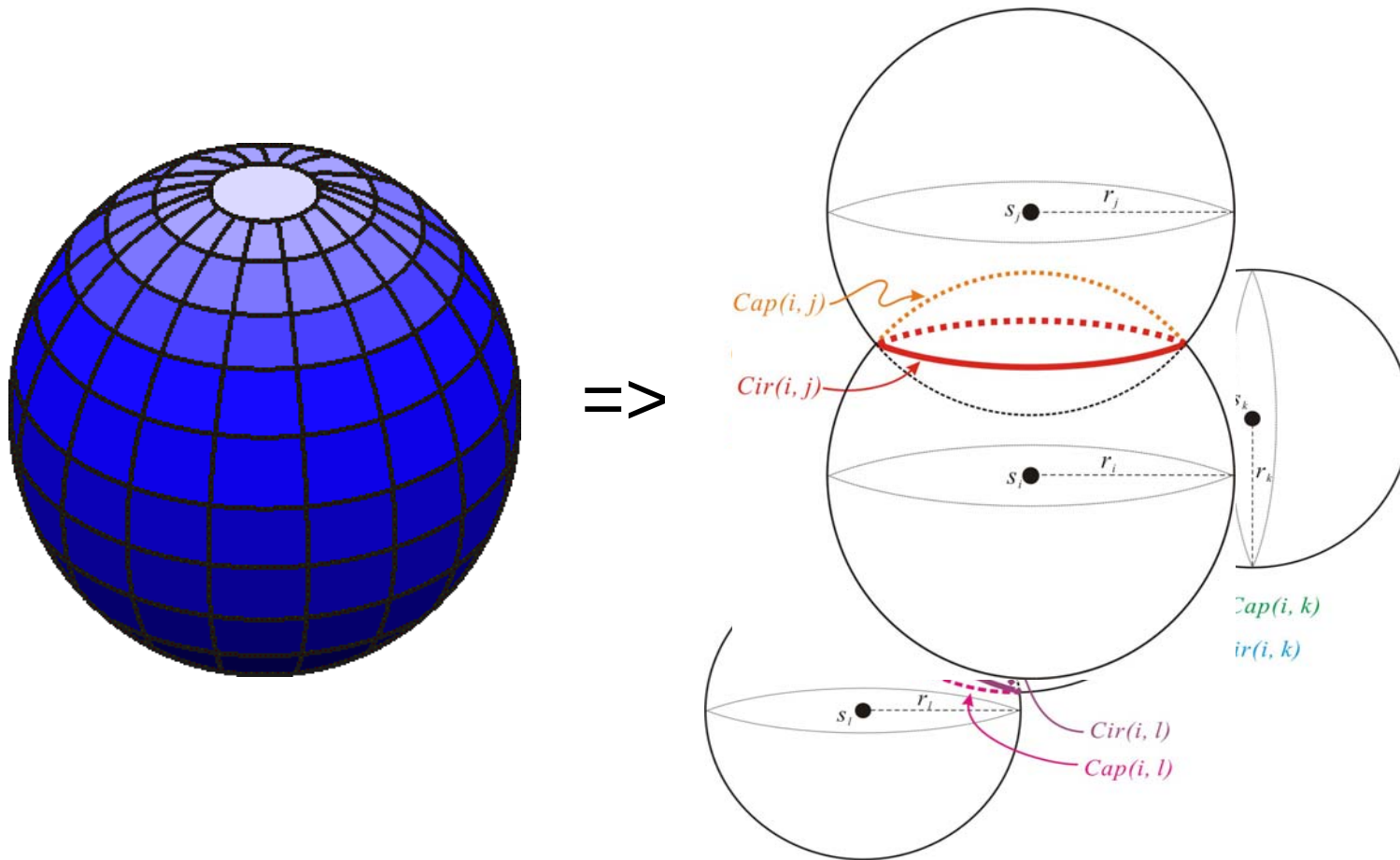
Reduction Technique (IV)

- $2D \Rightarrow 1D$
 - By stretching each circle on a 1D line, the level of circle coverage can be easily determined.
 - This can be done by our 2-D coverage solution.

Reduction Example

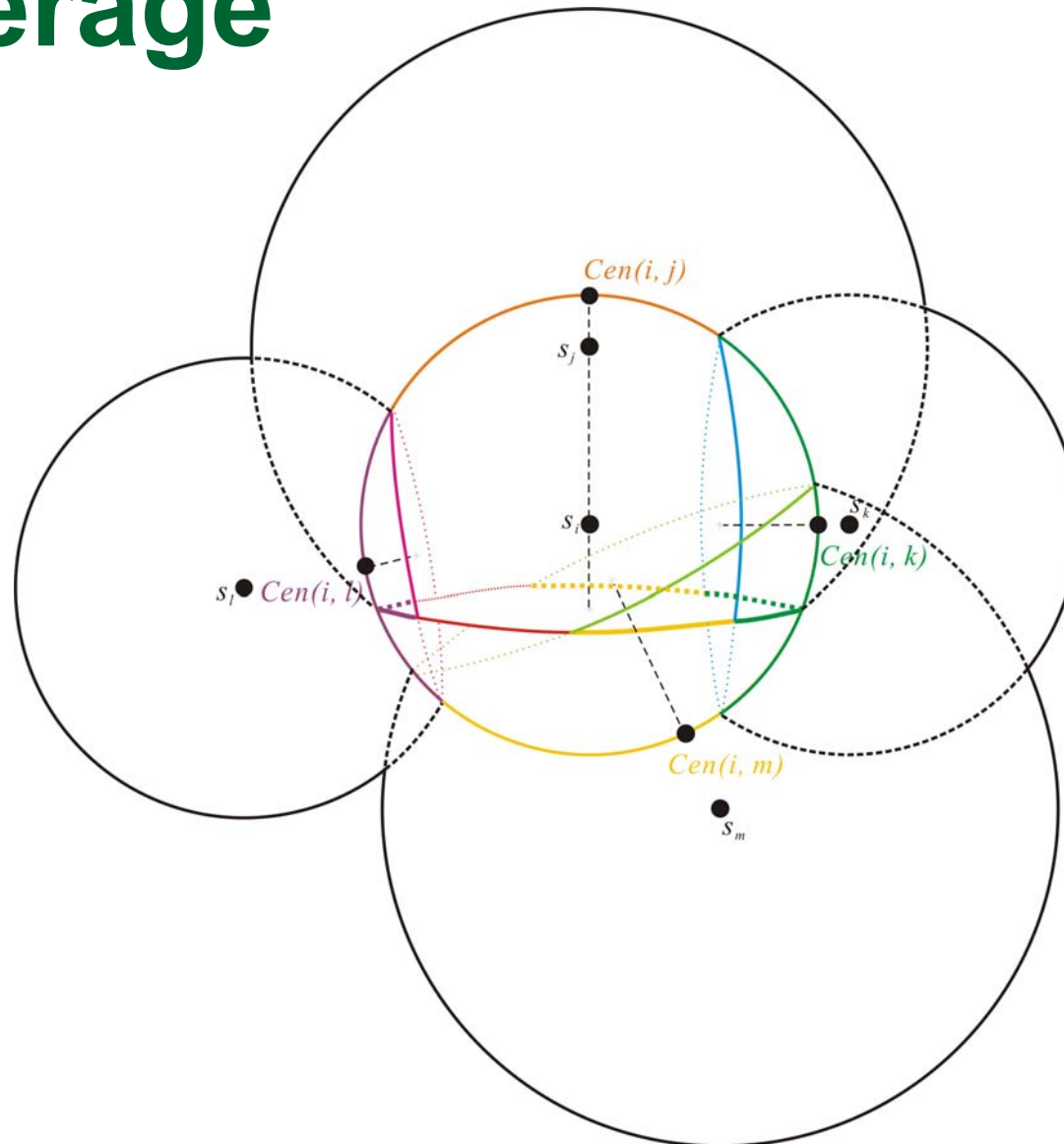


Reduction Example



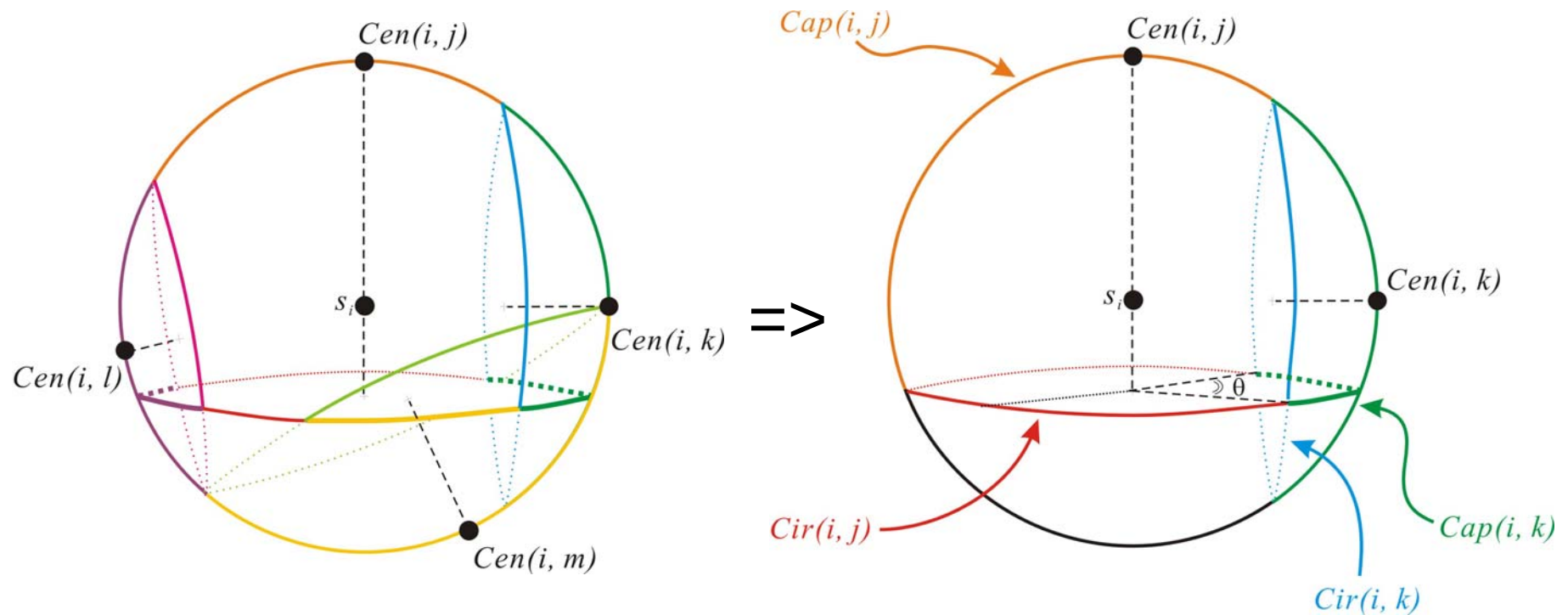
Calculating the Circle Coverage

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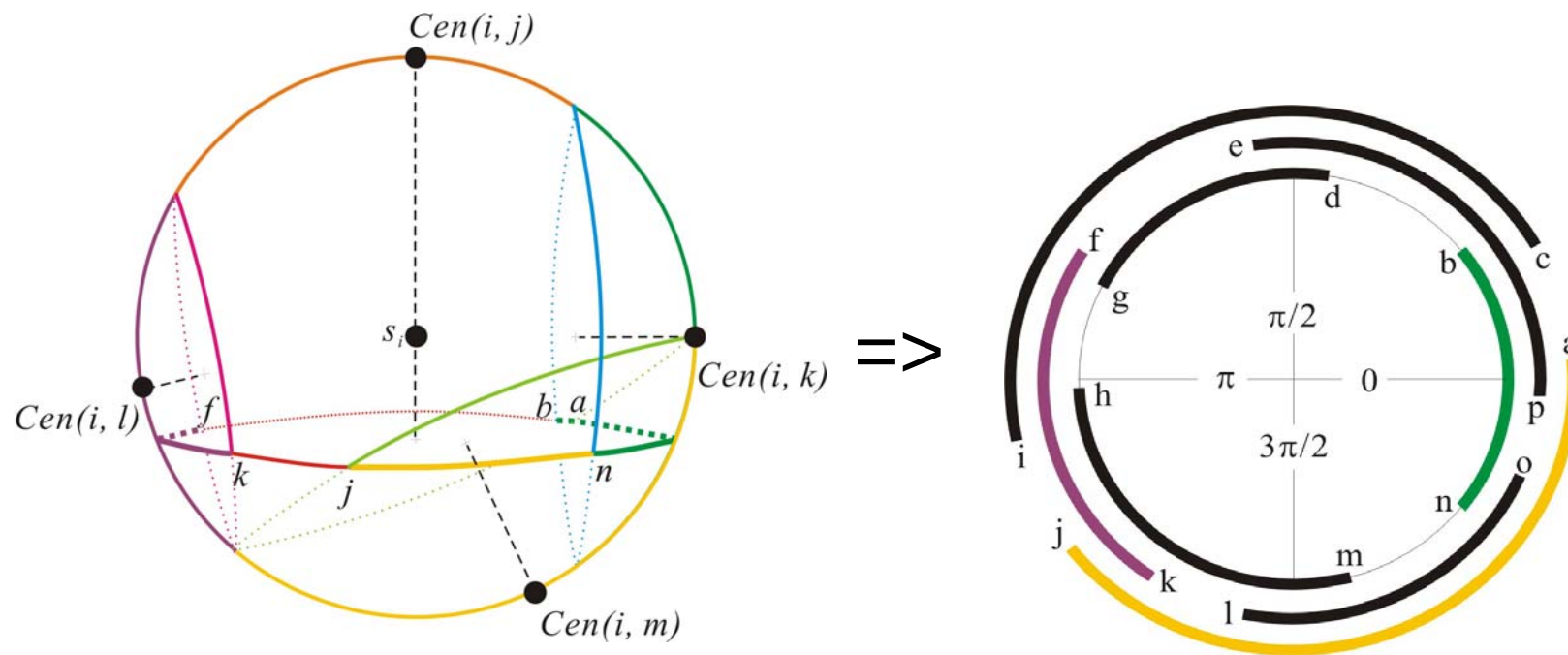


Calculating the Circle Coverage

34

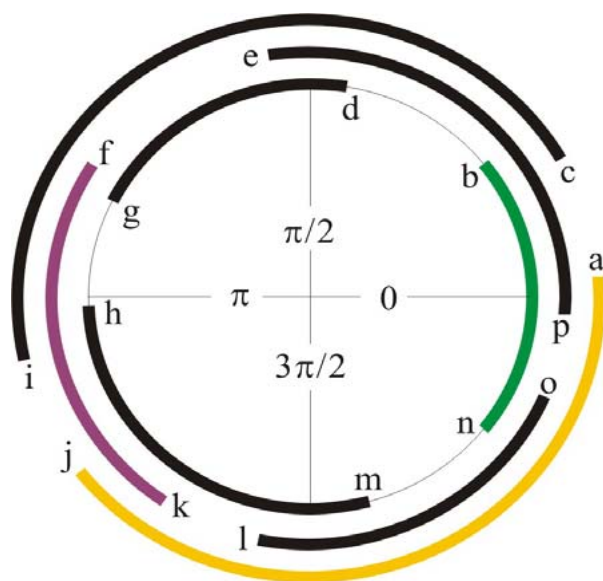


Calculating the Circle Coverage

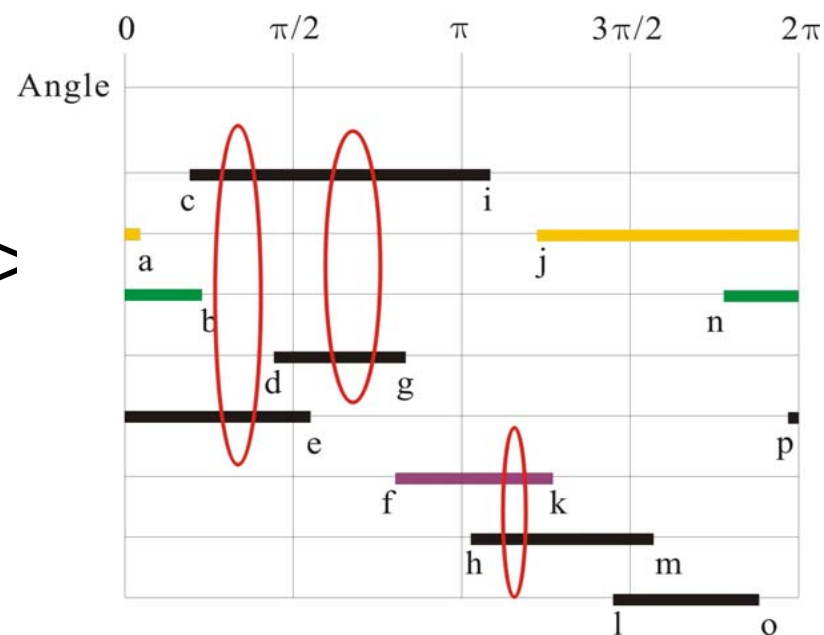


Calculating the Circle Coverage

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$\Rightarrow$



The Complete Algorithm

- Each sensor s_i independently calculates the **circle coverage** of each of the circle on its sphere.
 - **sphere coverage of s_i** =
 $\min \{ \text{circle coverage of all circles on } S_i \}$
- overall coverage = $\min \{ \text{sphere coverage of all sensors} \}$

Complexity

- To calculate the sphere coverage of one sensor:
 $O(d^2 \log d)$
 - d is the maximum number of neighbors of a sensor
- Overall: $O(nd^2 \log d)$
 - n is the number of sensors in this field

Efficient Placement and Dispatch of Sensors in a Wireless Sensor Network

Outline

- **Introduction**
- Sensor Placement
- Sensor Dispatch
- Conclusions

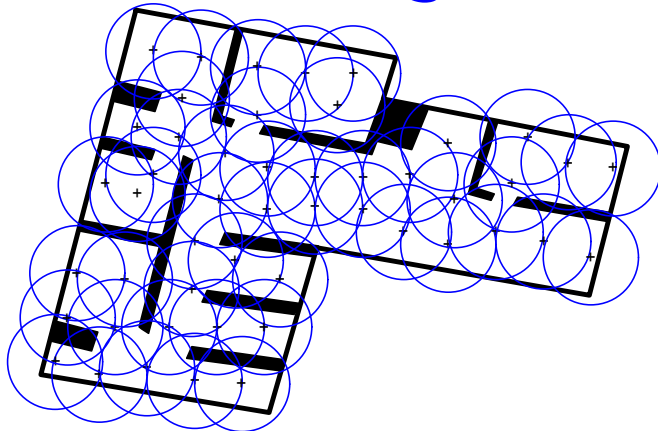
Introduction

- Wireless sensor networks (WSN)
 - Tiny, low-power devices
 - Sensing units, transceiver, actuators, and even mobilizers
 - Gather and process environmental information
- WSN applications
 - Surveillance
 - Biological detection
 - Monitoring

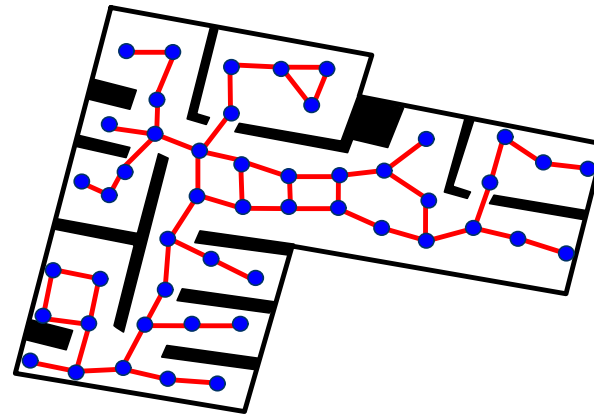


Introduction

- Sensor deployment is a critical issue because it affects the *cost* and *detection capability* of a wireless sensor network
- A good sensor deployment should consider both *coverage* and *connectivity*



Coverage



Connectivity

Review

- The *art gallery problem (AGP)* asks how to use a minimum set of guards in a polygon such that every point of the polygon is watched by at least one guard.
- However, the results cannot be directly applied to sensor deployment problem because
 - AGP typically assumes that a guard can watch a point as long as line-of-sight exists
 - Sensing distance of a sensor is normally *finite*
 - AGP does NOT address the communication issue between guards
 - Sensor deployment needs to address the *connectivity* issue

Two Issues in Sensor Deployment

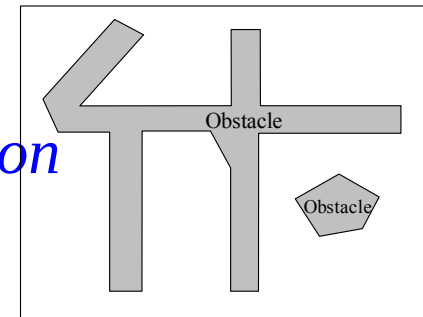
- Sensor placement problem:
 - Ask how to place the *least number of sensors* in a field to achieve desired coverage and connectivity properties.
- Sensor dispatch problem:
 - Assume that sensors are *mobilized*
 - Given a set of mobile sensors and an area of interest $\mathbf{I}$ inside the sensing field $\mathbf{A}$, to choose *a subset of sensors* to be delegated to $\mathbf{I}$ with *certain objective functions* such that the coverage and connectivity properties can be satisfied

Outline

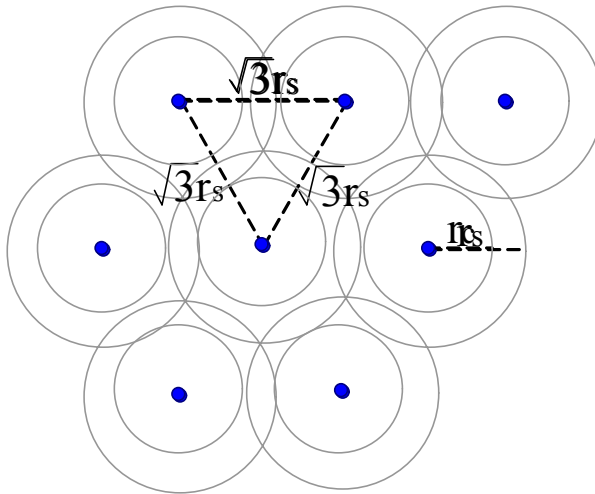
- Introduction
- **Sensor Placement**
- Sensor Dispatch
- Conclusions

Sensor Placement Problem

- Input: sensing field $\mathbf{A}$
 - $\mathbf{A}$ is modeled as an *arbitrary-shaped polygon*
 - $\mathbf{A}$ may contain several obstacles
 - Obstacles are also modeled by *polygons*
 - Obstacles do NOT partition $\mathbf{A}$
- Each sensor has a sensing distance r_s and communication distance r_c
 - But we do NOT restrict the relationship between r_s and r_c
- Our goal is to place sensors in $\mathbf{A}$ to ensure both sensing coverage and network connectivity *using as few sensors as possible*

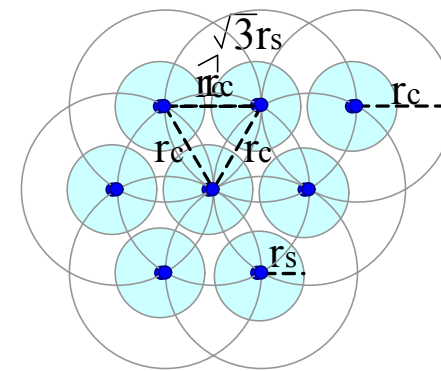


Two Intuitive Placements



Consider coverage first

Need to add extra sensors
to maintain **connectivity**
when $r_c < \sqrt{3}r_s$



Consider connectivity first

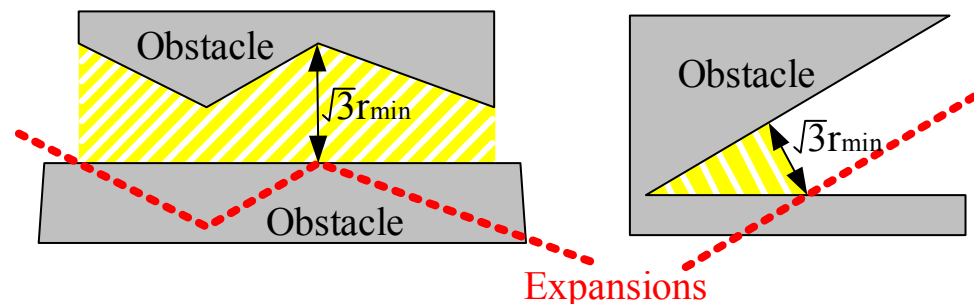
Need to add extra sensors
to maintain **coverage**
when $r_c > \sqrt{3}r_s$

Proposed Placement Algorithm

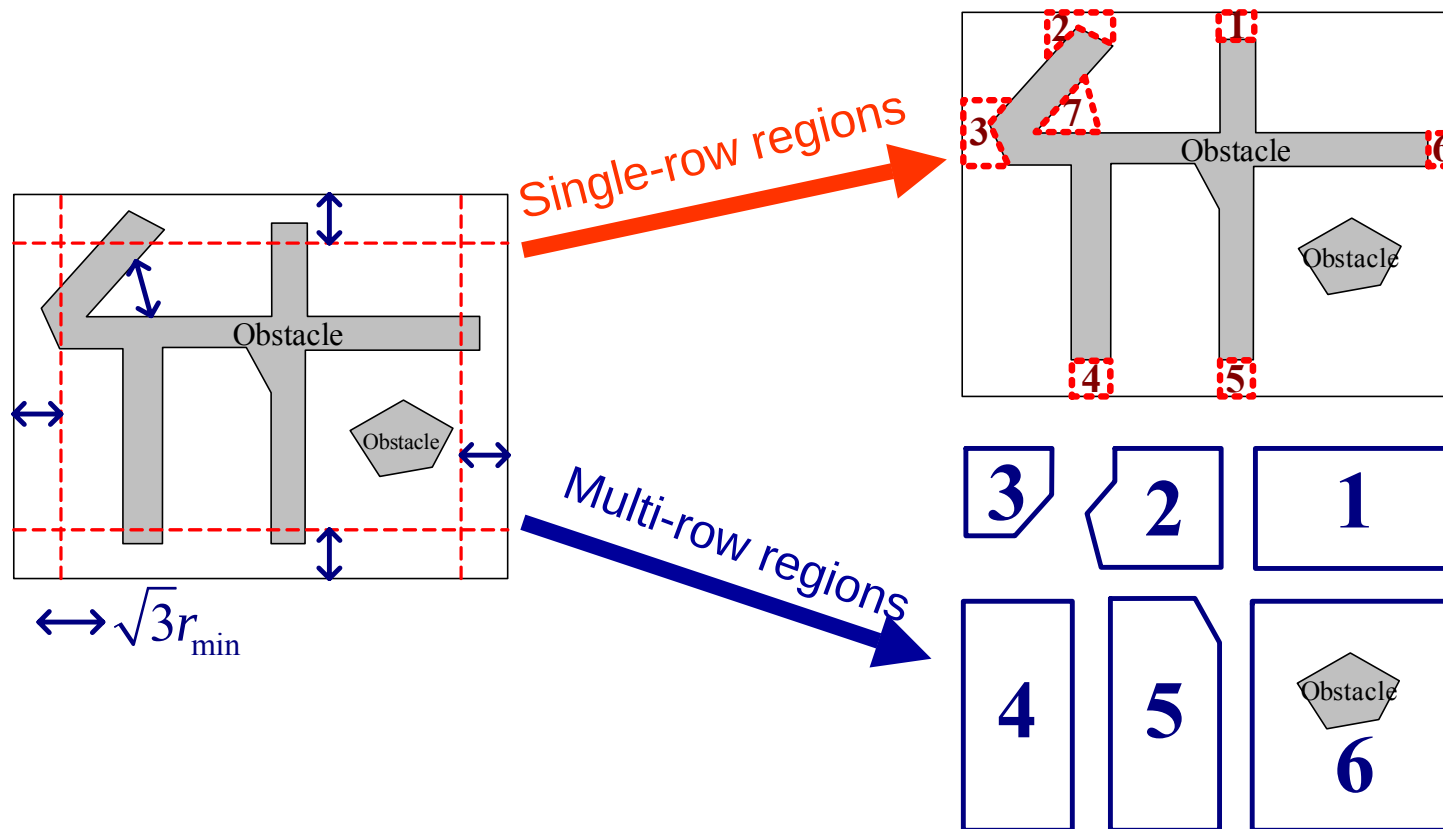
- Partition the sensing field $\mathbf{A}$ into two types of sub-regions:
 - *Single-row regions*
 - A belt-like area between obstacles whose width is NOT larger than $\sqrt{3}r_{\min}$, where $r_{\min} = \min(r_s, r_c)$
 - We can deploy *a sequence of sensors* to satisfy both coverage and connectivity
 - *Multi-row regions*
 - We need multi-rows sensors to cover such areas
 - Note: Obstacles may exist in such regions.

Step 1: Partition the Sensing Field

- From the sensing field $\mathbf{A}$, we identify all *single-row regions*
 - Expand the perimeters of obstacles *outwardly* and $\mathbf{A}$'s boundaries *inwardly* by a distance of $r_{\min}$
 - If the expansion overlaps with other obstacles, then we can take a *projection* to obtain single-row regions
- The remaining regions are multi-row regions.

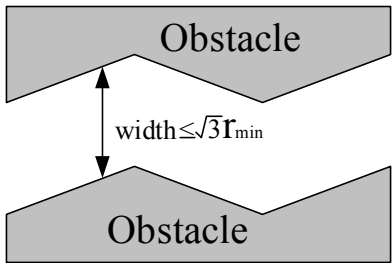
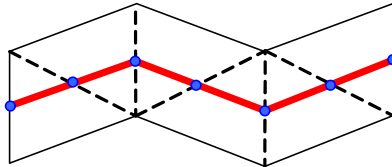
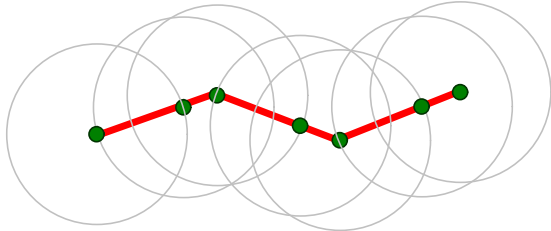
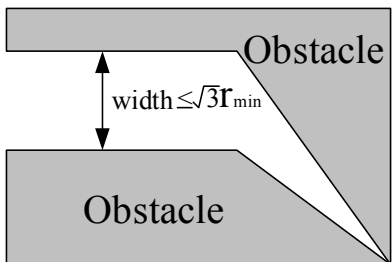
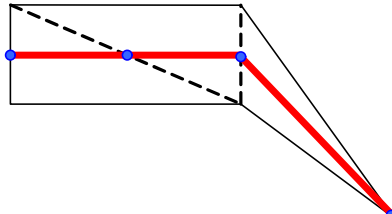
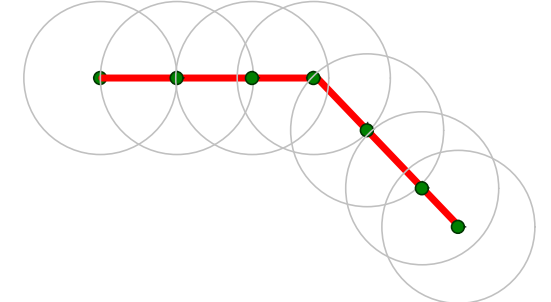


An Example of Partition



Step 2: Place Sensors in a Single-row Region

- Deploy sensors along the *bisector* of region

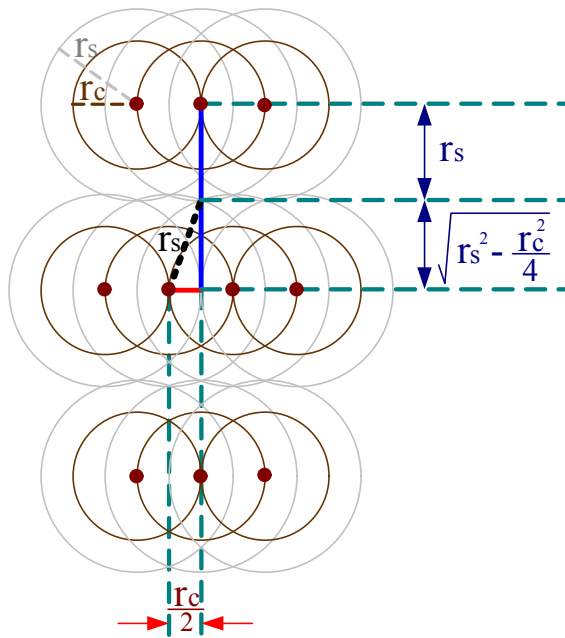
Case	Small Regions	Bisectors	Sensor Deployment
(a)			
(b)			

Step 3: Place Sensors in a Multi-row Region

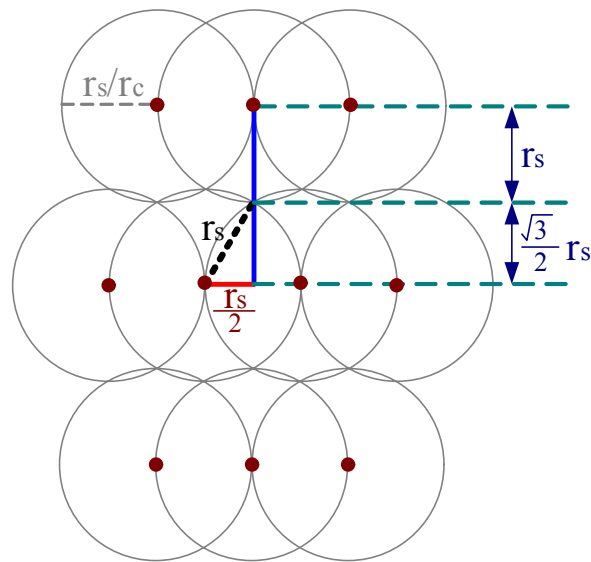
- We first consider a 2D plane without boundaries & obstacles
 - Deploy sensors *row by row*
 - A row of sensors needs to guarantee *coverage and connectivity*
 - Adjacent rows need to guarantee *continuous coverage*
- Case 1: $r_c \leq \sqrt{3}r_s$
 - Sensors on each row are separated by r_c
 - Adjacent rows are separated by $r_s + \sqrt{r_s^2 - \frac{r_c^2}{4}}$
- Case 2: $r_c > \sqrt{3}r_s$
 - Each sensor is separated by $\sqrt{3}r_s$

Case 1:

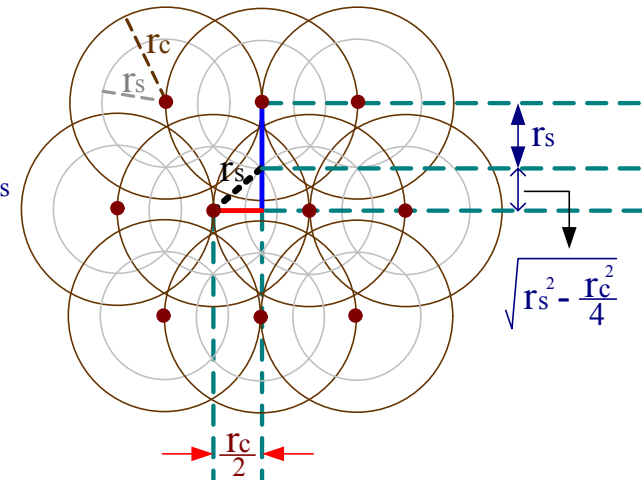
$$r_c \leq \sqrt{3}r_s$$



$$r_s > r_c$$



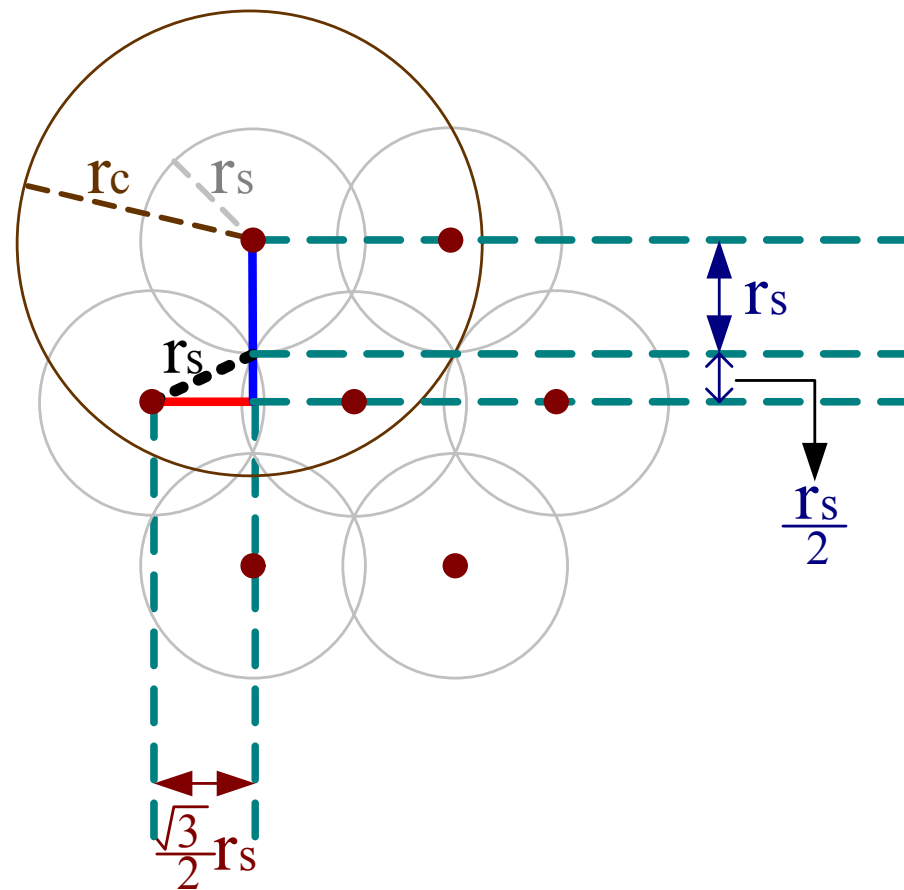
$$r_s = r_c$$



$$r_s < r_c \leq \sqrt{3}r_s$$

Case 2:

$$r_c > \sqrt{3}r_s$$

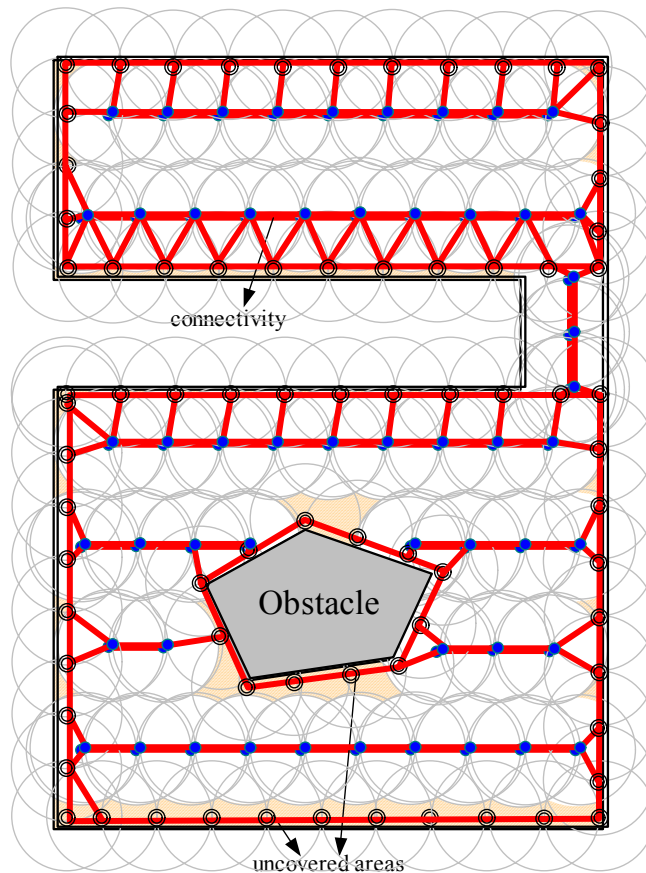


Refined Step 3:

- For a multi-row region with boundaries and obstacles,
 - We can place sensors one by one according to the following locations (if it is not inside an obstacle or outside the region)

Neighbor	$r_c \leq \sqrt{3}r_s$	$r_c > \sqrt{3}r_s$
N_1	$(x + r_c, y)$	$(x + \sqrt{3}r_s, y)$
N_2	$(x + \frac{r_c}{2}, y - \sqrt{r_s^2 - \frac{r_c^2}{4}} - r_s)$	$(x + \frac{\sqrt{3}r_s}{2}, y - \frac{3r_s}{2})$
N_3	$(x - \frac{r_c}{2}, y - \sqrt{r_s^2 - \frac{r_c^2}{4}} - r_s)$	$(x - \frac{\sqrt{3}r_s}{2}, y - \frac{3r_s}{2})$
N_4	$(x - r_c, y)$	$(x - \sqrt{3}r_s, y)$
N_5	$(x - \frac{r_c}{2}, y + \sqrt{r_s^2 - \frac{r_c^2}{4}} + r_s)$	$(x - \frac{\sqrt{3}r_s}{2}, y + \frac{3r_s}{2})$
N_6	$(x + \frac{r_c}{2}, y + \sqrt{r_s^2 - \frac{r_c^2}{4}} + r_s)$	$(x + \frac{\sqrt{3}r_s}{2}, y + \frac{3r_s}{2})$

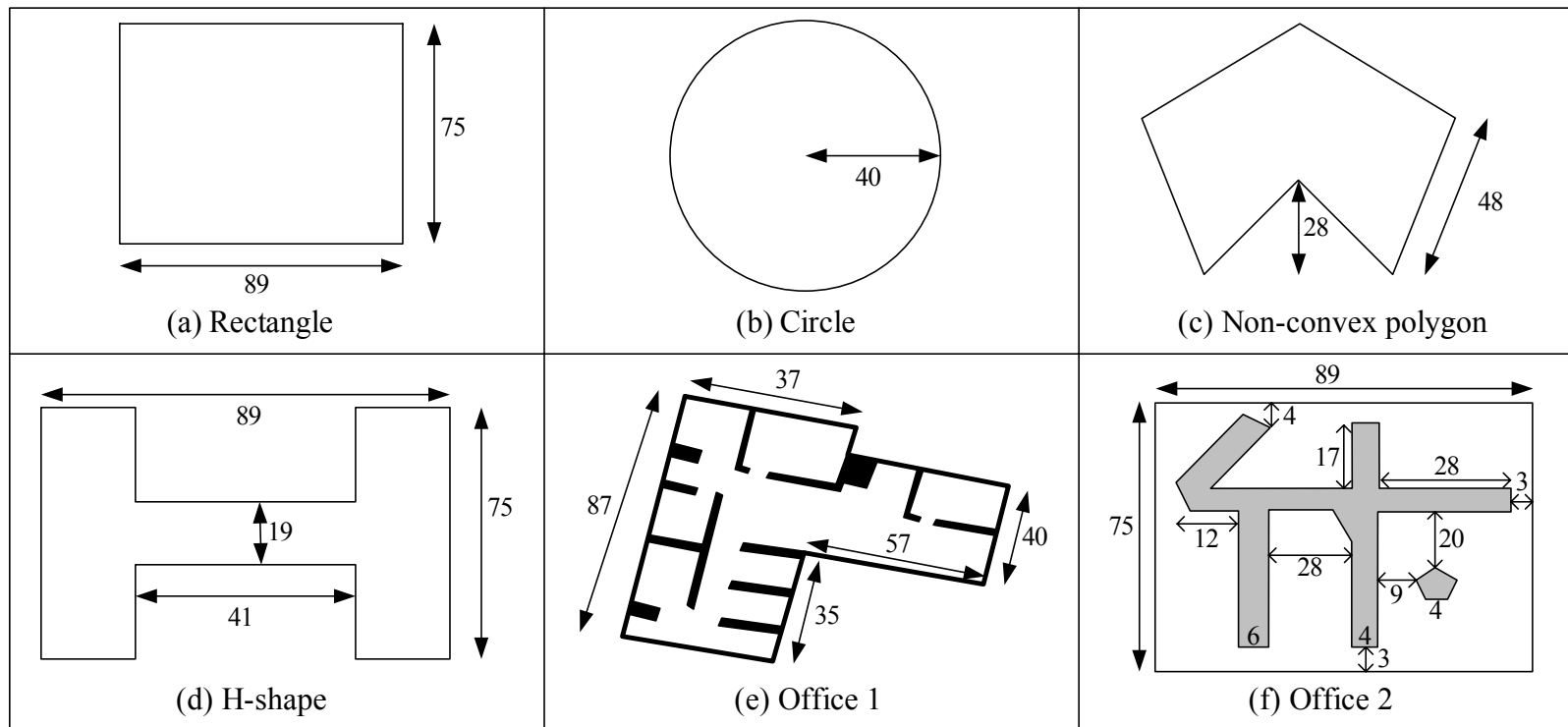
Step 4:



- Three unsolved problems
 - Some areas *near the boundaries* are uncovered
 - Need extra sensors between *adjacent rows* to maintain connectivity when $r_c < \sqrt{3}r_s$
 - Connectivity to *neighboring regions* needs to be maintained
- Solutions
 - Sequentially place sensors along the boundaries of the regions and obstacles

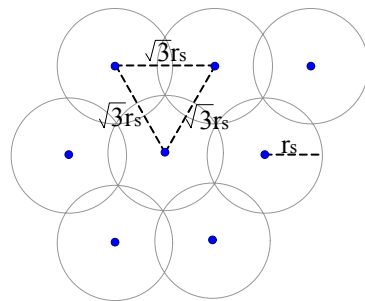
Simulation Results

- Sensing fields

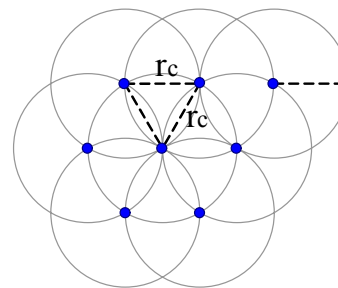


Simulation Parameters

- We use $(r_s, r_c) = (7,5), (5,5), (3.5,5), (2,5)$ to reflect the four cases $r_s > r_c, r_s = r_c, r_s < r_c \leq \sqrt{3}r_s, \sqrt{3}r_s < r_c$
- Comparison metric
 - Average number of sensors used to deploy
 - Compare with two deployment methods

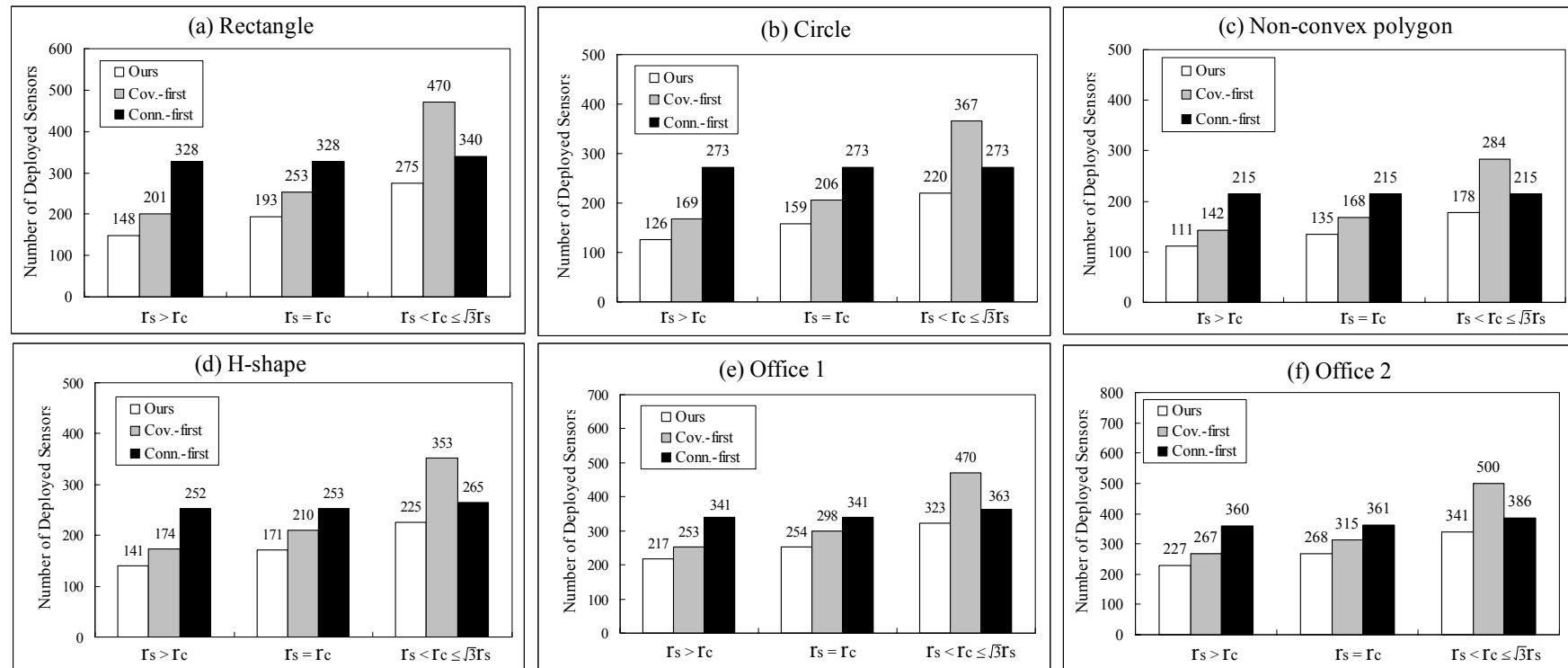


Coverage-first

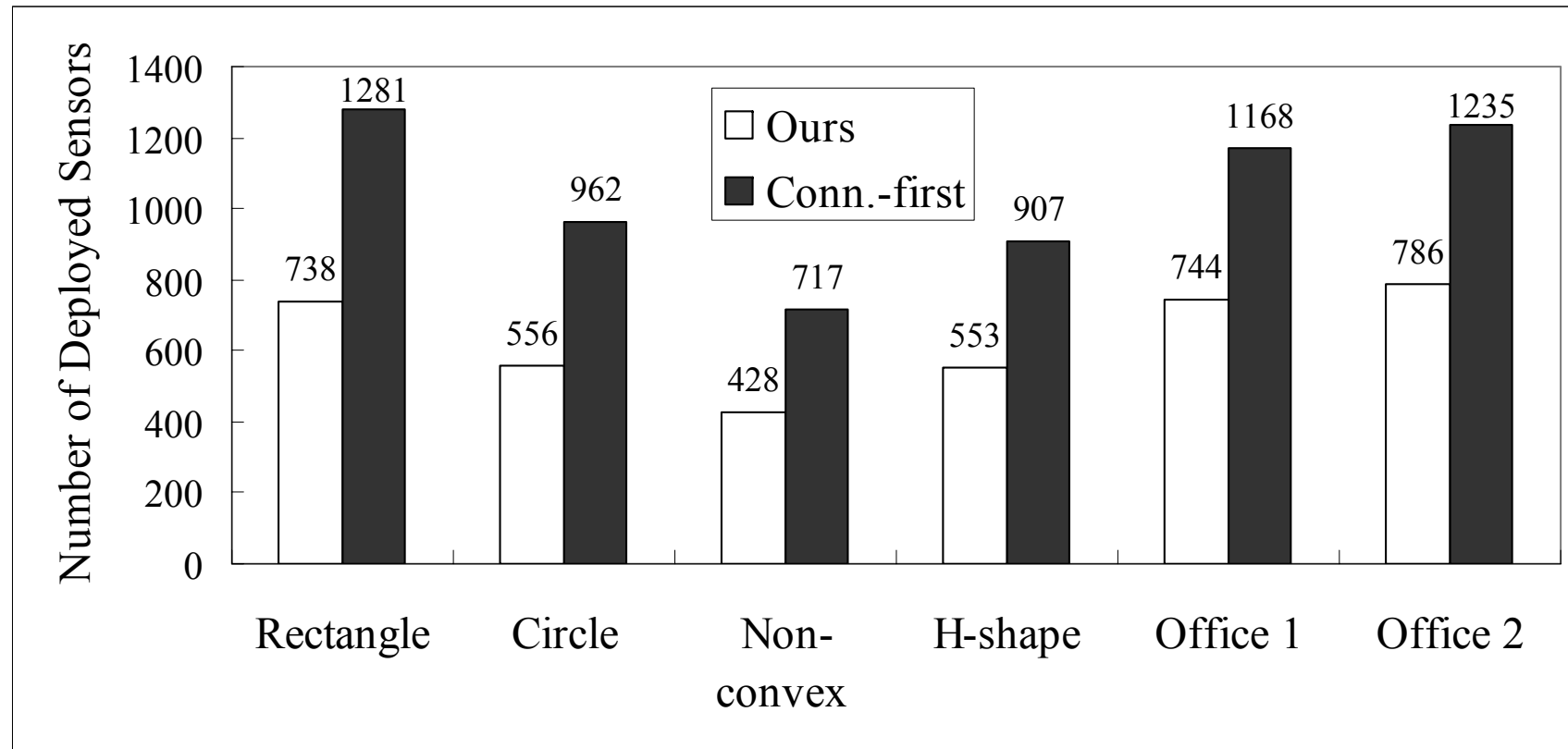


Connectivity-first

Simulations (r_s vs. r_c)



Simulations (Shapes of A)



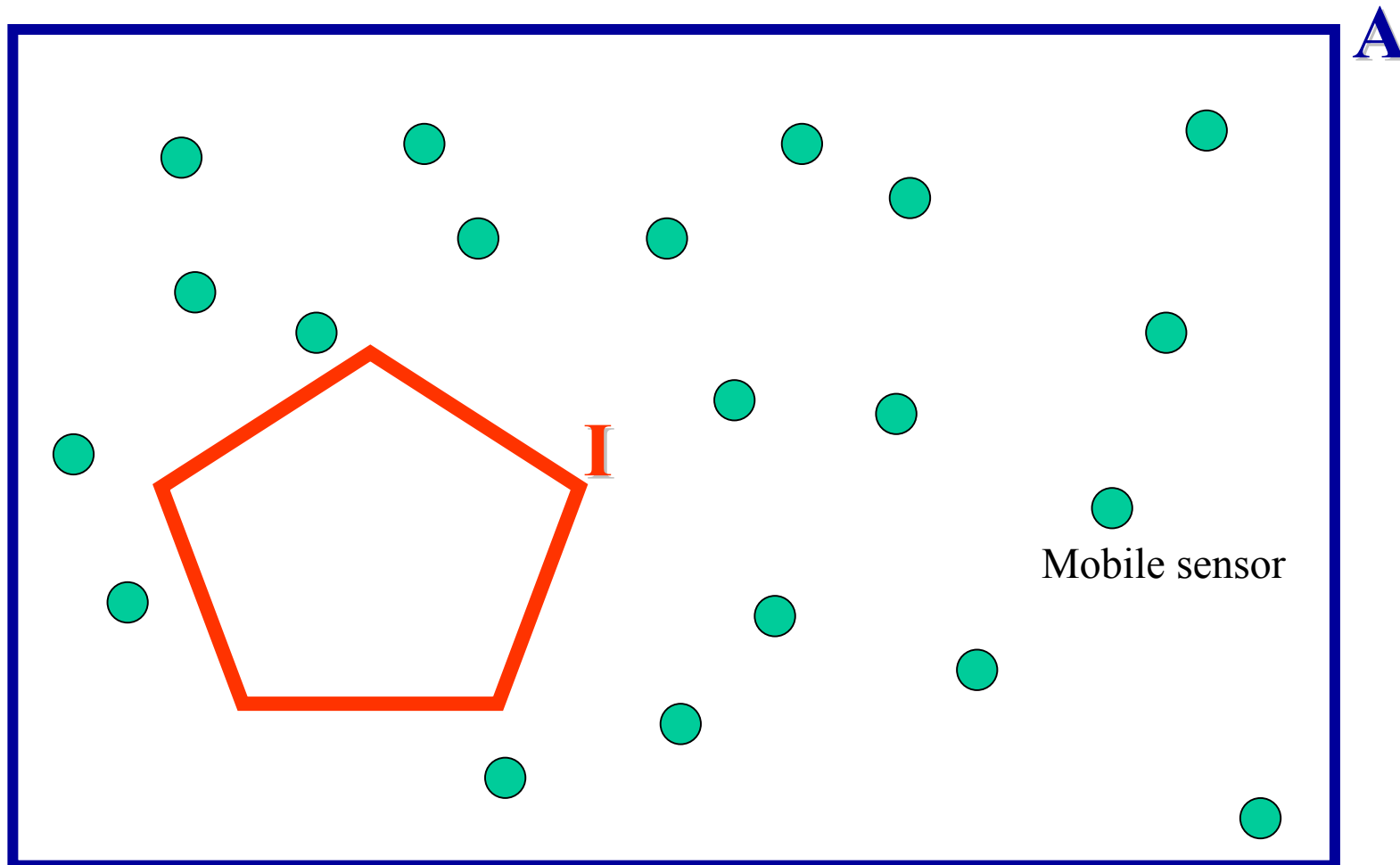
Outline

- Introduction
- Sensor Placement
- **Sensor Dispatch**
- Conclusions

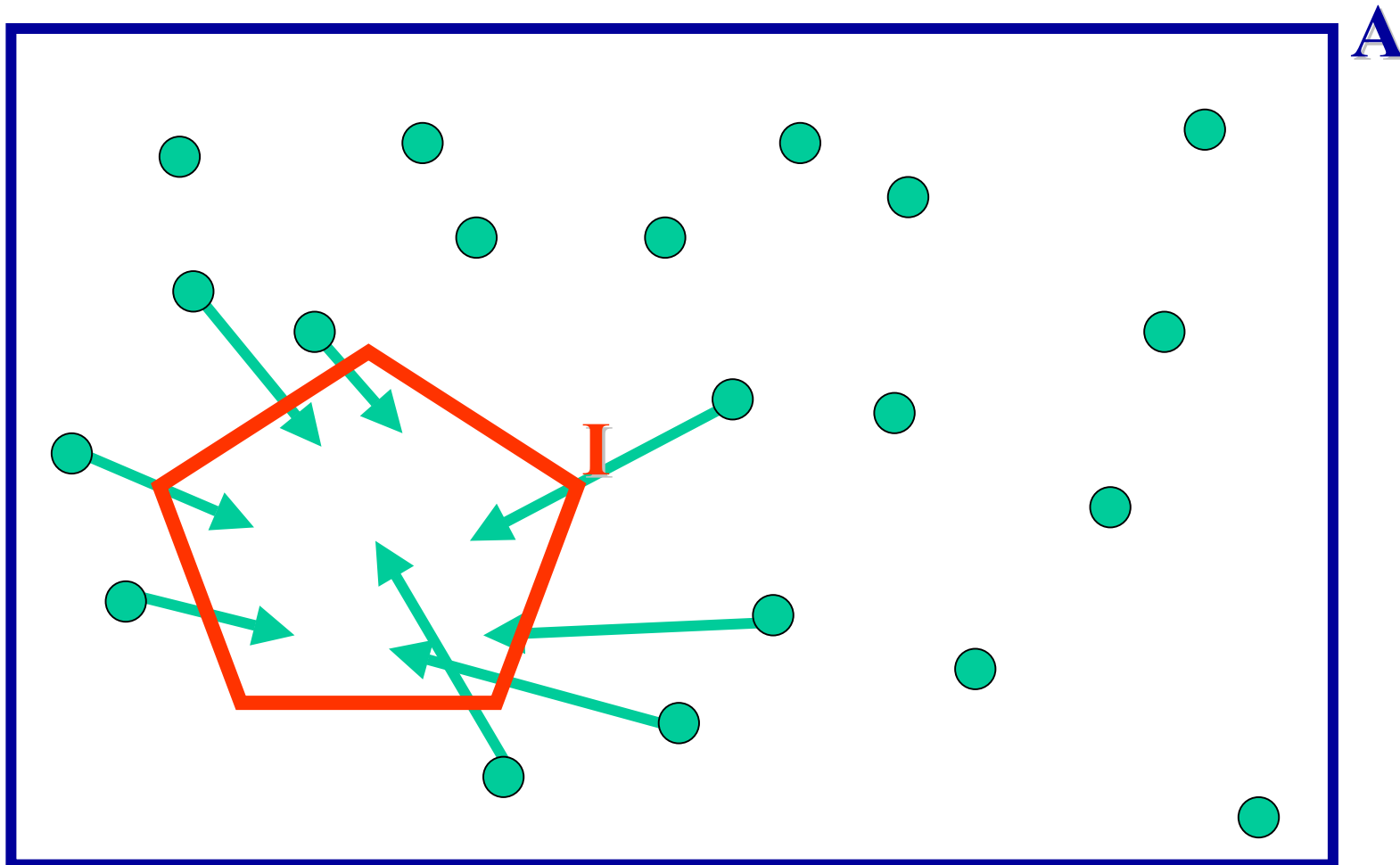
Problem Definition

- We are given
 - A sensing field $\mathbf{A}$
 - An area of interest $\mathbf{I}$ inside $\mathbf{A}$
 - A set of mobile sensors $\mathbf{S}$ resident in $\mathbf{A}$
- The sensor dispatch problem asks how to find a *subset* of sensors $\mathbf{S}'$ in $\mathbf{S}$ to be moved to $\mathbf{I}$ such that after the deployment, $\mathbf{I}$ satisfies *coverage* and *connectivity* requirements and the movement cost satisfies some *objective functions*.

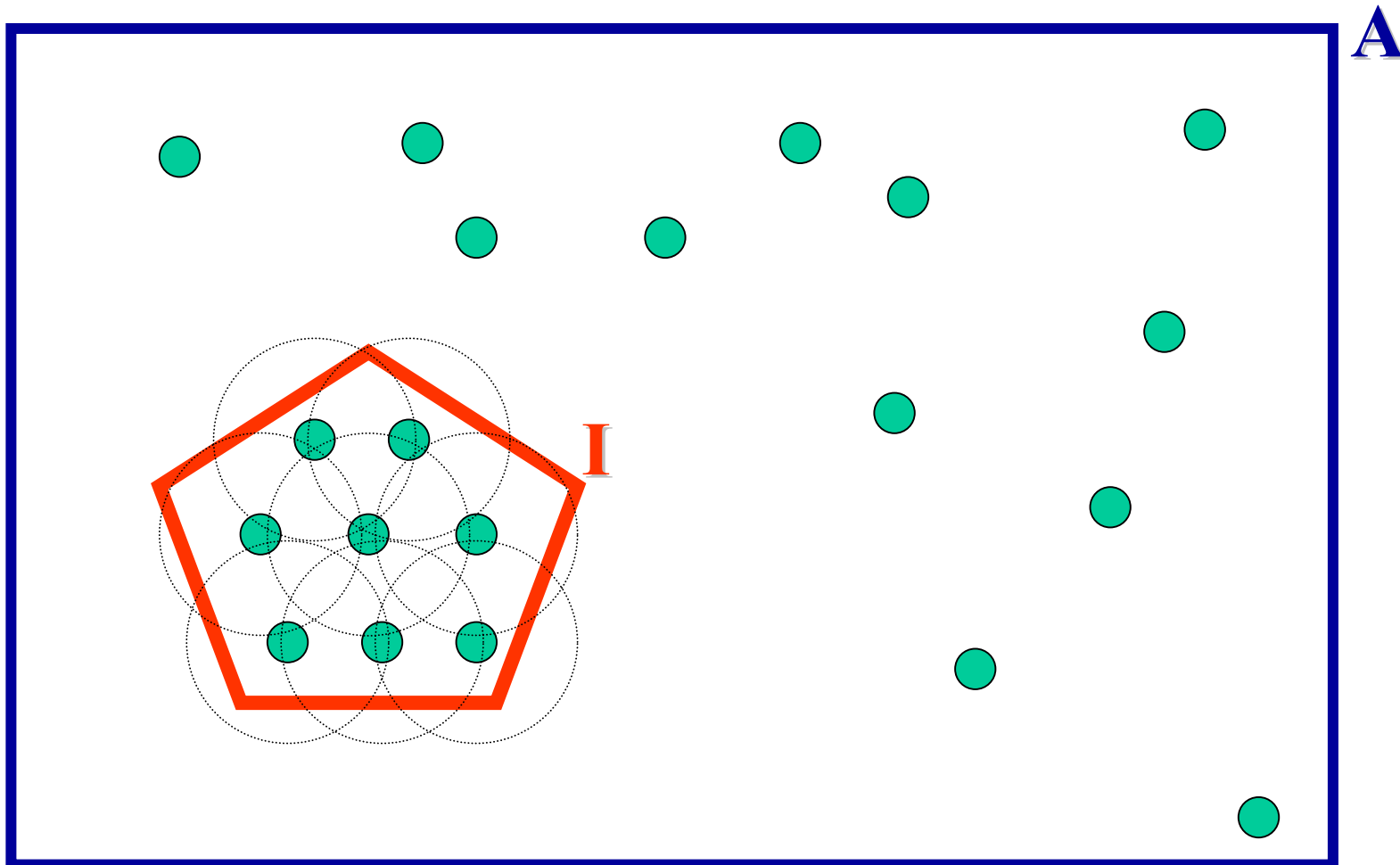
Example



Example



Example



Two Objective Functions

- Minimize the *total energy consumption* to move sensors

$$\min \sum_{i \in S'} \Delta_m \times d_i$$

- Δ_m : unit energy cost to move a sensor in one step
- d_i : the distance that sensor i is to be moved

- Maximize the *average remaining energy* of sensors in S' after the movement

$$\max \frac{\sum_{i \in S'} (e_i - \Delta_m \times d_i)}{|S'|}$$

- e_i : initial energy of sensor i

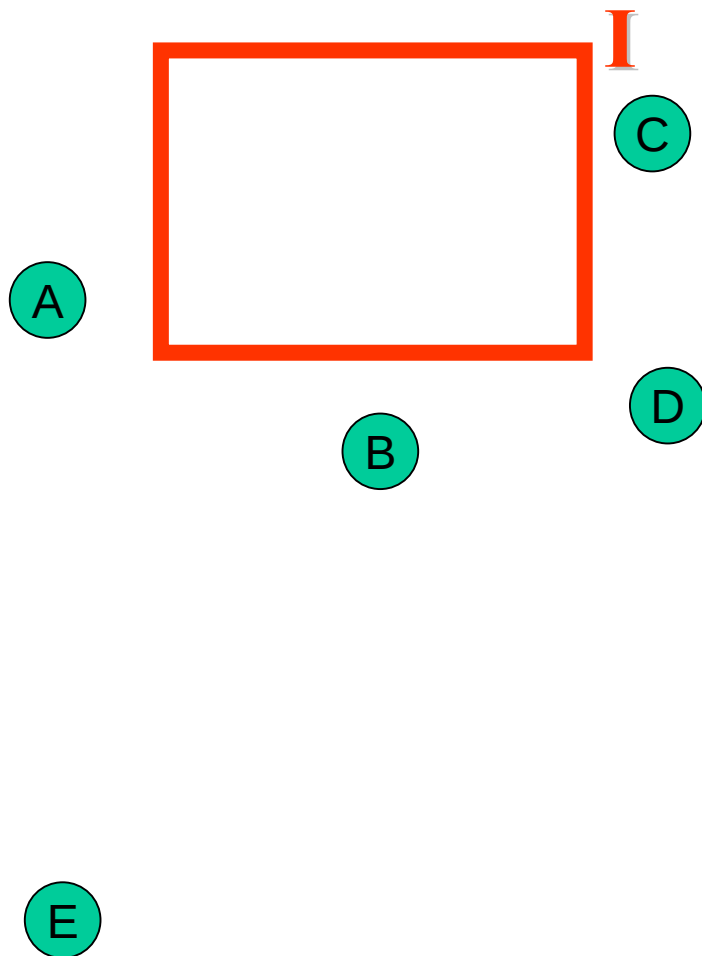
Proposed Dispatch Algorithm (I)

- Run any sensor **placement algorithm** on $\mathbf{I}$ to get the target locations $L = \{(x_1, y_1), \dots, (x_m, y_m)\}$
- For each sensor $s_i \in S$, determine the **energy cost** $c(s_i, (x_j, y_j))$ to move s_i to each location (x_j, y_j)
 - $c(s_i, (x_j, y_j)) = \Delta_m \times d(s_i, (x_j, y_j))$
- Construct a **weighted complete bipartite graph** $G = (S \cup L, S \times L)$ such that the weight of each edge is
 - $w(s_i, (x_j, y_j)) = -c(s_i, (x_j, y_j))$, if objective function (1) is used; or as
 - $w(s_i, (x_j, y_j)) = e_i - c(s_i, (x_j, y_j))$, if objective function (2) is used

Proposed Dispatch Algorithm (II)

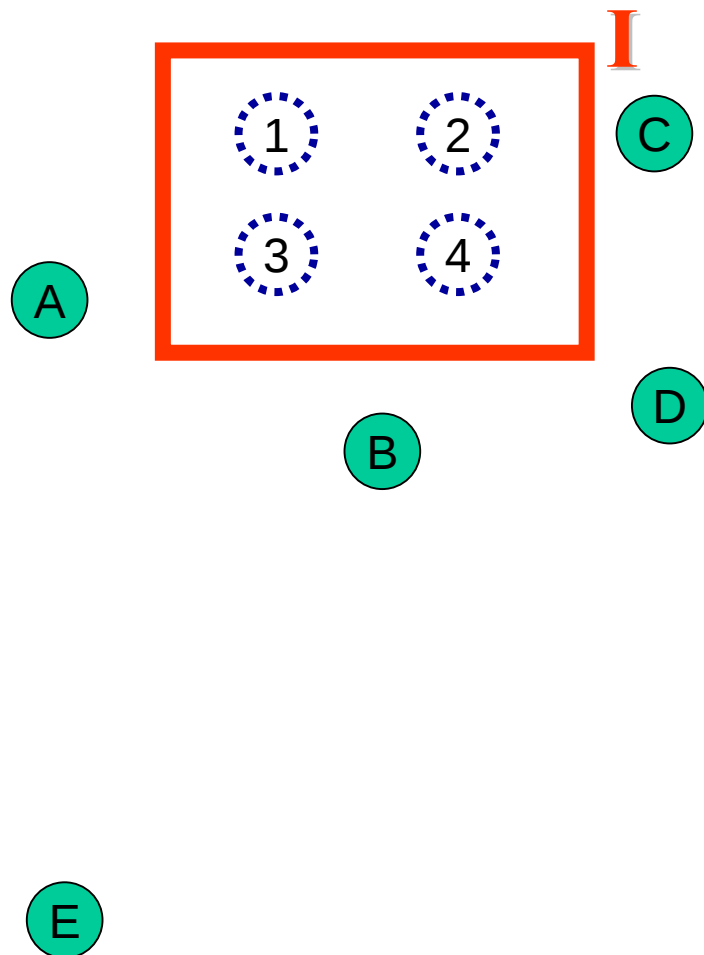
- Construct a new graph $\hat{G} = (S \cup L \cup \hat{L}, S \times \{L \cup \hat{L}\})$ from G , where $\hat{L}$ is a set of $|S| - |L|$ elements, each called a *virtual location*. The weights of edges incident to $\hat{L}$ are set to w_{min} , where $w_{min} = \{\text{min. weight in } G\} - 1$.
- Find the maximum-weight perfect-matching M on graph $\hat{G}$ by using the *Hungarian method*.
- For each edge $c(s_i, (x_j, y_j))$ in M such that $(x_j, y_j) \notin \hat{L}$, move sensor s_i to location (x_j, y_j) via the shortest path.
 - If $e_i - c(s_i, (x_j, y_j)) \leq 0$, it means that we do not have sufficient energy to move all sensors. Then the algorithm terminates.

An Example of Dispatch



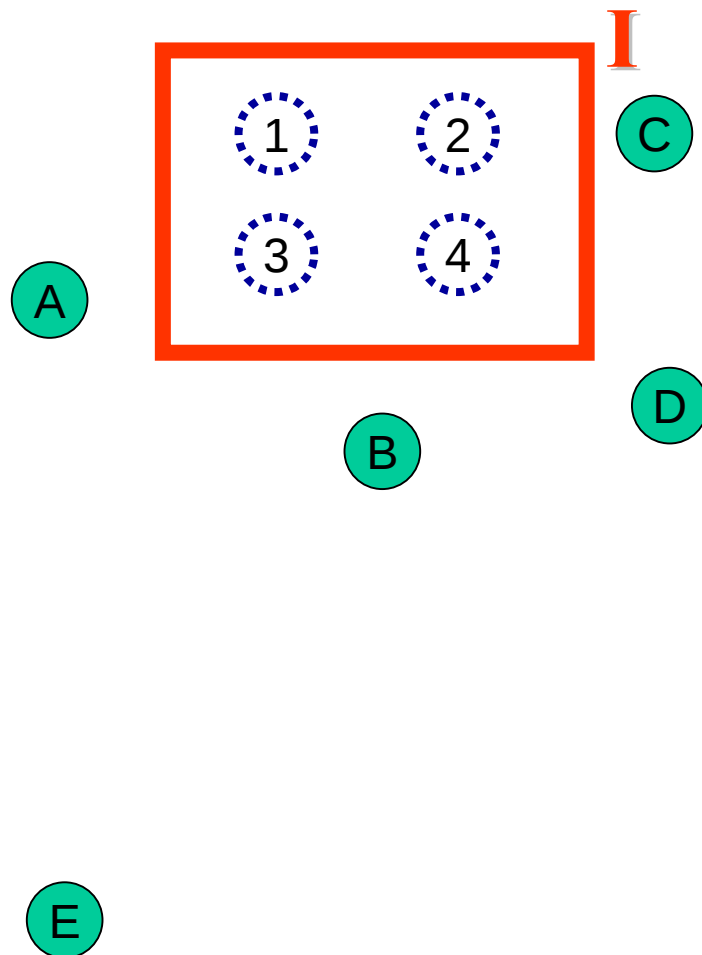
- Initially, there are five mobile sensors A, B, C, D, and E

An Example of Dispatch



- Run sensor placement algorithm on **I** to get the target locations
 $L = \{(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)\}$

An Example of Dispatch

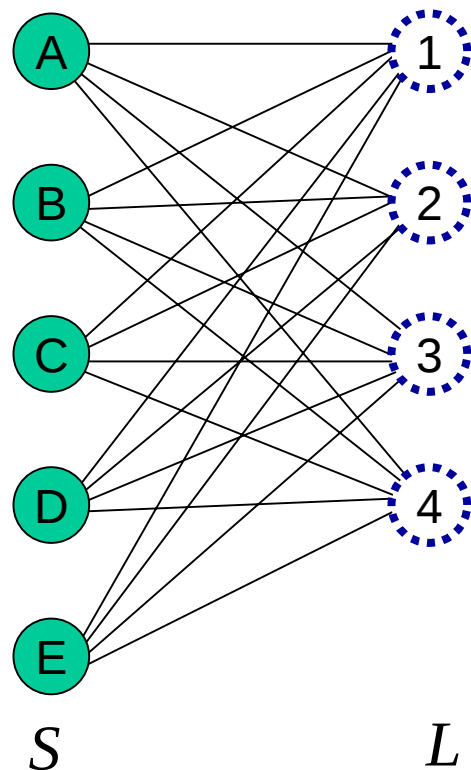


- Compute energy cost (assume $\Delta_m = 1$)

$c(A, (x_1, y_1)) = 9$	$c(A, (x_2, y_2)) = 12$
$c(A, (x_3, y_3)) = 8$	$c(A, (x_4, y_4)) = 11$
$c(B, (x_1, y_1)) = 11$	$c(B, (x_2, y_2)) = 11$
$c(B, (x_3, y_3)) = 9$	$c(B, (x_4, y_4)) = 9$
$c(C, (x_1, y_1)) = 10$	$c(C, (x_2, y_2)) = 6$
$c(C, (x_3, y_3)) = 11$	$c(C, (x_4, y_4)) = 8$
$c(D, (x_1, y_1)) = 14$	$c(D, (x_2, y_2)) = 13$
$c(D, (x_3, y_3)) = 12$	$c(D, (x_4, y_4)) = 10$
$c(E, (x_1, y_1)) = 33$	$c(E, (x_2, y_2)) = 35$
$c(E, (x_3, y_3)) = 30$	$c(E, (x_4, y_4)) = 31$

An Example of Dispatch

- Construct the weighted complete bipartite graph G and assign weight on each edge

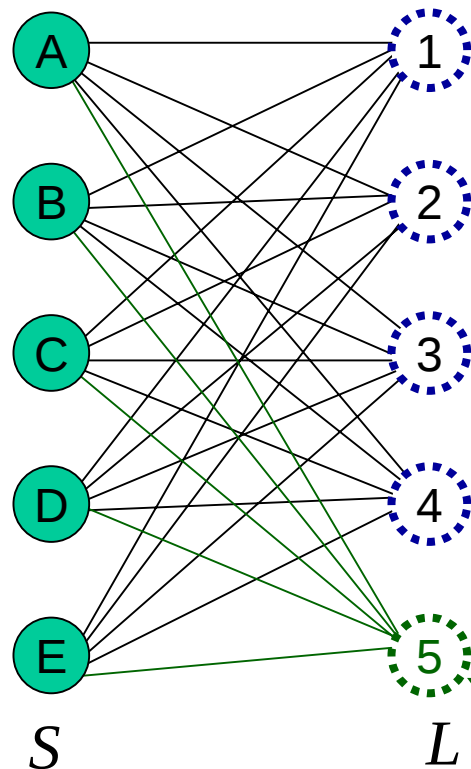


Weights of edges (assume that all sensors have the same initial energy 40 & 1st objective function is used)

	A	B	C	D	E
1	31	29	30	26	7
2	28	29	36	27	5
3	32	31	29	38	10
4	29	31	32	30	9

An Example of Dispatch

- Construct the new graph $\hat{G}$ from G by adding $|S|-|L|$ virtual locations



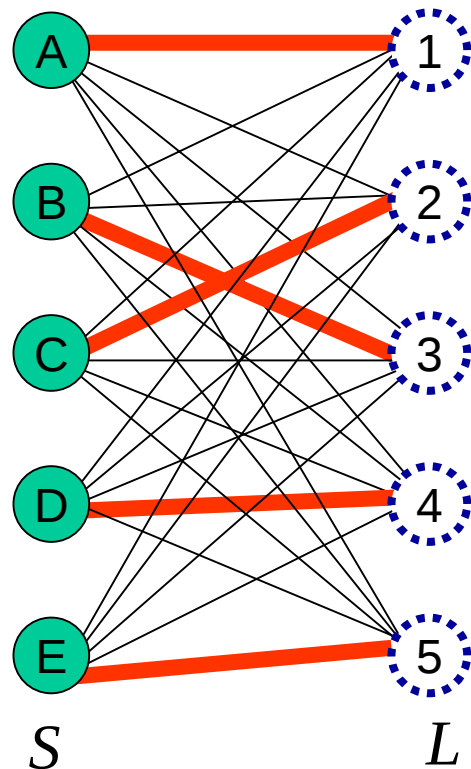
Weights of edges

	A	B	C	D	E
1	31	29	30	26	7
2	28	29	34	27	5
3	32	31	29	28	10
4	29	31	32	30	9
5	4	4	4	4	4

Min.

An Example of Dispatch

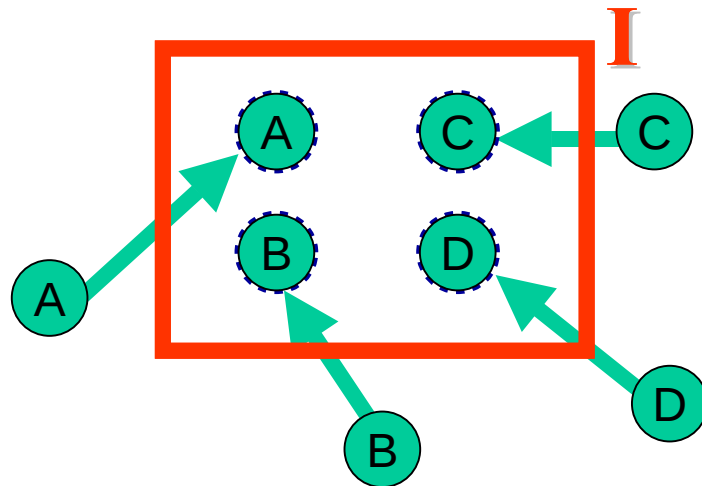
- Use the Hungarian method to find a maximum-weighted perfect-matching M



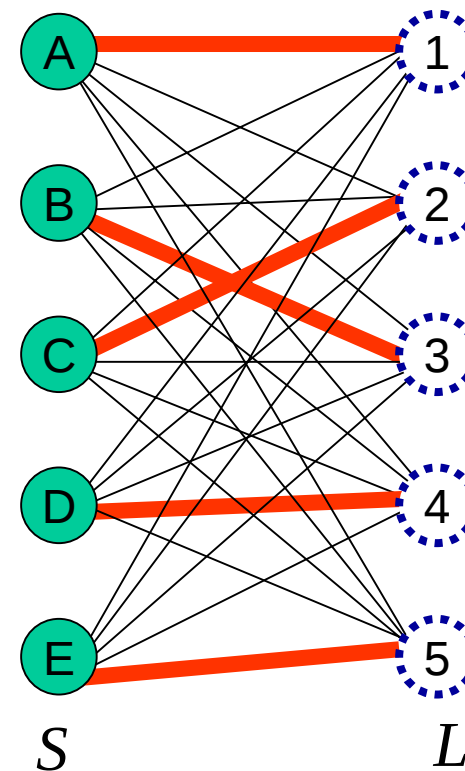
Weights of edges

	A	B	C	D	E
1	31	29	30	26	7
2	28	29	34	27	5
3	32	31	29	28	10
4	29	31	32	30	9
5	4	4	4	4	4

An Example of Dispatch



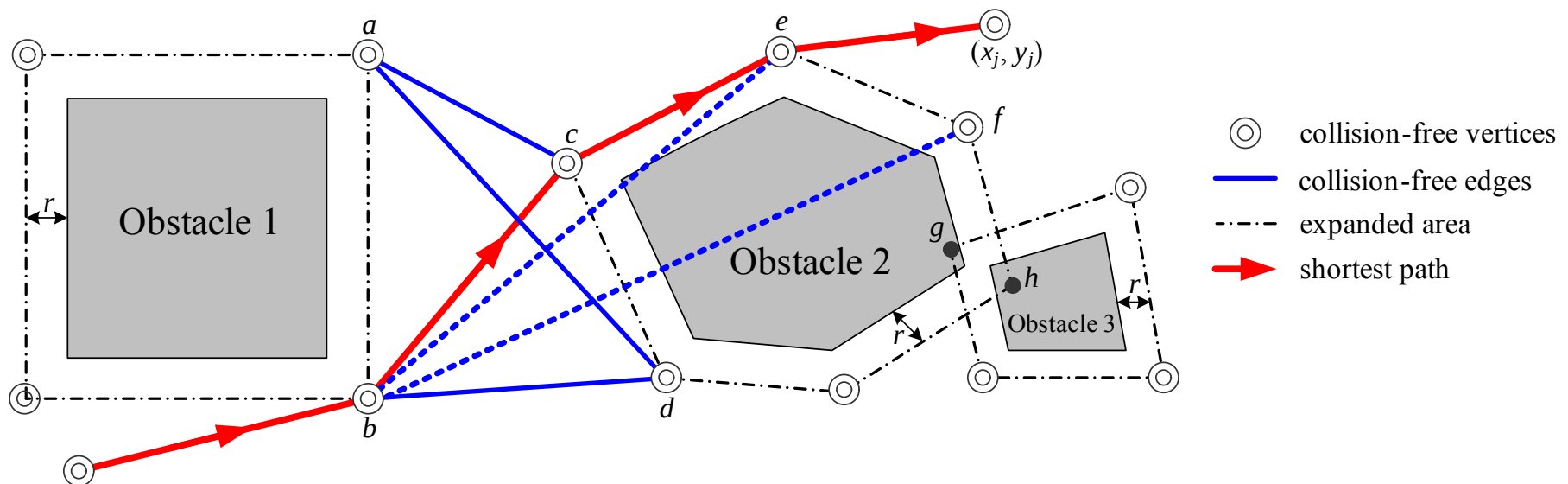
- Move sensors to the target locations



E Do not move

Find the Shortest Distance $d(s_p, (x_j, y_j))$

- Find *collision-free shortest* path
 - A sensor is modeled as a *circle* with a radius r
 - Expand the perimeters of obstacles by the distance of r to find the *collision-free vertices*.
 - Connect all pairs of vertices, as long as the corresponding edges do not cross any obstacle.
 - Using *Dijkstra's algorithm* to find the shortest path.



Find the Maximum-Weight Perfect-Matching

Definition 1. Given $\hat{G} = (S \cup L \cup \hat{L}, S \times \{L \cup \hat{L}\})$, a *feasible vertex labeling* of $\hat{G}$ is a real-valued function f on $S \cup L \cup \hat{L}$ such that for all $s_i \in S$ and $(x_j, y_j) \in \{L \cup \hat{L}\}$,

$$f(s_i) + f((x_j, y_j)) \geq w(s_i, (x_j, y_j)).$$

Definition 2. Given a feasible vertex labeling of $\hat{G}$, an *equality subgraph* $\hat{G}_f = (S \cup L \cup \hat{L}, E_f)$ is the subgraph of $\hat{G}$ in which E_f contains all edges $(s_i, (x_j, y_j))$ in $\hat{G}$ such that

$$f(s_i) + f((x_j, y_j)) = w(s_i, (x_j, y_j)).$$

Theorem 1. Let f be a feasible vertex labeling of $\hat{G}$ and M be a perfect matching of $\hat{G}_f$, then M is a maximum-weight perfect matching of $\hat{G}$.

The Hungarian Method

Step 1: Find a maximum matching M in $\hat{G}_f$. If M is perfect, we find out the solution and the method terminates. Otherwise, there must be an unmatched vertex $s_i \in S$. We then assign two sets $A = \{s_i\}$ and $B = \emptyset$.

Step 2: In the graph $\hat{G}_f$, if $N_{\hat{G}_f}(A) \neq B$, where $N_{\hat{G}_f}(A)$ is the set of vertices in $\{L \cup \hat{L}\}$ that are adjacent to the vertices in A , then go to step 3. Otherwise, we set

$$\alpha = \min_{s_i \in A, (x_j, y_j) \in \{L \cup \hat{L}\} - B} \{f(s_i) + f((x_j, y_j)) - w(s_i, (x_j, y_j))\},$$

and construct a new labeling f' for $\hat{G}$ by

$$f'(v) = \begin{cases} f(v) - \alpha & \text{for } v \in A \\ f(v) + \alpha & \text{for } v \in B \\ f(v) & \text{otherwise} \end{cases}.$$

Then we replace f by f' , reconstruct the equality subgraph $\hat{G}_{f'}$, and go to step 1. Note that we have to satisfy the conditions of $\alpha > 0$ and $N_{\hat{G}_{f'}}(A) \neq B$; otherwise, we need to reselect another α value that can satisfy the above conditions.

Step 3: Choose a vertex (x_l, y_l) in $N_{\hat{G}_f}(A)$ but not in B . If (x_l, y_l) is matched with $s_k \in S$ in M , then we update $A = A \cup \{s_k\}$ and $B = B \cup \{(x_l, y_l)\}$, and go back to step 2.

Time complexity

- The time complexity of our sensor dispatch algorithm is $O(mnk^2 + n^3)$
 - m : number of target locations in $\mathbf{I}$
 - n : number of mobile sensors
 - k : number of vertices of the polygons of all obstacles and $\mathbf{I}$