Paging strategy optimization in personal communication systems

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Mobility tracking is concerned with finding a mobile subscriber (MS) within the area serviced by the wireless network. The two basic operations for tracking an MS, location updating and paging, constitute additional load on the wireless network. The total cost of updating and paging can be minimized by optimally dividing the cellular area into location registration (LR) areas. In current systems broadcast paging messages are sent within the LR area to alert the MS of an incoming call. In this paper we propose a *selective paging* strategy which uses the MS mobility and call patterns to minimize the cost of locating an MS within an LR area subject to a constraint on the delay in locating the MS.

1. Introduction

The wireless personal communications system (PCS) architecture consists of a group of base stations (BS) covering the service area and interconnected by a fixed backbone network. The coverage area of the BS, determined by the terrain and the radio propagation characteristics, is referred to as a cell. Each cell contains a number of mobile subscribers which connect to the BS via wireless links. To provide ubiquitous location independent communications, the cellular coverage of the BSs overlap to provide the MS with a continuous coverage during a call and in the idle state. In the idle state the MS monitors the control (paging) channel for incoming calls or order messages from the BS. To be able to alert the MS of incoming calls or to send order messages to a specific MS, the system has to be able to track the movements of the MS within the system. To facilitate the tracking of the moving MS, the PCS network is partitioned into location registration (LR) areas. Each LR area may contain one or more cells.

It is the task of the mobility manager (MM) of the system to handle all issues associated with MS location tracking. The mobility manager resides in the mobile switching center (MSC). The MSC is the central coordinating element for all cell sites and LR areas within its service area. The MM contains two databases to facilitate the tracking of MSs; the home location register (HLR) and the visitor location register (VLR). The HLR contains database information on the MSs whose primary subscription is within this area. The VLR, on the other hand, contains database information for all MSs currently active within the service area of the MM.

When an MS enters a new location area it performs a location update (LU) on the access channels of one of the BSs in the new LR area. The VLR is updated to reflect the new location information for that MS and the HLR of that MS is updated to reflect the new location of the VLR. When an incoming call is destined for the MS, the HLR and the VLR are used to find the the LR area of the MS. Paging messages are broadcast by every BS in the LR area to alert the MS [6].

In [2] we introduced the mobility tracking problem in wireless communications. The two fundamental operations associated with mobility tracking, location update (LU) and paging, contribute to the total signaling cost on the control channels. Minimizing the total cost of signaling by minimizing the LU cost and paging cost constitutes contradicting goals. In [2] we bound the cost of paging, by keeping the number of cells in the LR area constant, and optimized the LR area design to minimize the LU cost per MS. In this paper we address the second fundamental operation in mobility tracking, the paging strategy.

The paging strategy is used by the network to alert the MS in the LR area in which it is registered. This is performed whenever there is a need to establish communications with a particular MS. In the current paging strategy paging messages are broadcast by each BS in the LR area where the MS is registered. When the MS receives the paging signal it starts an access procedure identical to that employed when the MS initiates a call [10]. Under this strategy the overhead due to transmissions of paging messages is maximum, since every cell in the LR area broadcasts the paging message. On the other hand, the delay associated with locating the MS is minimized. All the cells in the LR area search for the MS at the same time. In [3] another simple strategy is used for cell sites arranged in a ring topology: Upon the arrival of an incoming call for the MS, the system first pages the LU cell where the MS last reported. If the MS is not found there, then the system sequentially searches for the MS in cells on either side of the LU cell until the MS is reached. Under this strategy the cost associated with the transmission of paging messages is decreased, however the delay associated with locating the MS in the LR area is increased. If searching for an MS in a particular cell takes, on the average, d time units, the system must wait at least d units after paging a cell before paging the next cell.

There is a tradeoff between the paging cost and the delay associated with locating an MS, is demonstrated by the above two simple strategies. Bandwidth is a scarce resource and the blocking of users from the system due to bandwidth unavailability is much more undesirable from the user's and the operator's viewpoint than the delay of incoming calls reaching the user. However, the delay in reaching the user should not be increased to the extent that it degrades the quality of service below an acceptable limit.

The above two paging strategies are static schemes that do not exploit the call statistics and mobility pattern of the MSs. Using information on the mobility and call patterns of the MS, one may attempt to page the BSs inside the LR area in such a way that the signaling cost of paging and the delay associated with locating the MS are minimized. However, since these are contradicting goals, our approach is to bound one of the variables and to minimize the other. In our formulation we bound the delay cost and introduce a new *selective paging* scheme to minimize the paging signaling cost for a particular MS call statistics and mobility patterns in an LR area of given size and shape.

2. Selective paging strategy

We propose a new dynamic selective paging scheme to minimize the cost of paging. The *selective paging* scheme partitions the LR area, of size k cells, into N partitions, $P_1, P_2, \dots, P_N$, where $N \leqslant k$. A partition P_i contains n_i cells of the LR area A. The partitions are mutually exclusive and collectively exhaustive. Thus,

$$\forall (i,j) \in A, \quad \text{if } (i,j) \in P_l \text{ then } (i,j) \notin P_m \ \forall m \neq l,$$
$$\text{where} \quad m \in \{1, \dots, N\} \quad \text{and} \quad \sum_{q=1}^{N} n_q = k.$$

Partition P_1 contains the n_1 cells of the LR area that the MS is most likely to be located at the time of arrival of the incoming paging call. P_2 contains the next most likely n_2 cells and so on. The *selective paging* scheme pages first the cells in partition P_1. If the MS is not found there, the cells in partition P_2 are paged and so on until the MS is found. Searching for an MS in a partition is assumed to take d time units. If the MS is not found in the partition within d time units, the MS is assumed to be located in another partition. *Selective paging* is a dynamic paging strategy that exploits the statistics of incoming calls to the MS and the mobility characteristics in the LR area to optimally partition the LR area to minimize the paging cost subject to a constraint on the delay of the incoming calls reaching the MS. As the processing capability of the system increases, it would be possible to design per-user partitions, however, for current systems MSs with similar incoming call rates and mobility patterns may be grouped together and the partitions are designed per group. The *selective paging* shall be demonstrated for a MS which is representative of its group.

The *selective paging* strategy maps the cells inside the LR area A into the probability line (figure 1) at the time of arrival of the incoming call. This mapping as will be seen later depends on many factors, such as the mobility pattern of the MS in the LR area, the speed profile of the MS depending on traffic characteristics in the LR area, and the MS incoming call statistics. Define the following:

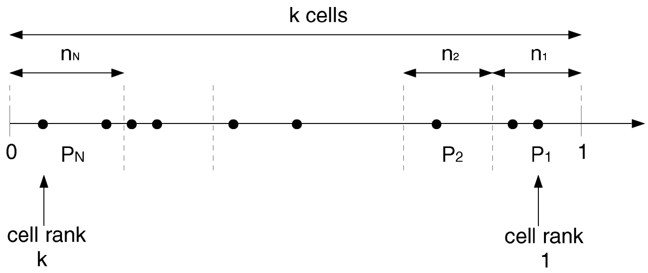

Figure 1. The probability line at time of arrival of incoming call.

- $\Pr(P_i)$ – probability that the MS is located in partition P_i at the time of arrival of the incoming call.
- $\Pr(\text{MS in } (l,m))$ – probability that the MS is located in the cell with coordinates (l,m) at the time of arrival of the incoming call.

Thus,

$$\Pr(P_i) = \sum_{\forall (l,m) \in P_i} \Pr\bigl(\text{MS in } (l,m)\bigr) \tag{1}$$

and $\sum_{i=1}^{N} \Pr(P_i) = 1$. Then the average number of cells paged in the LR area $\overline{n}_{\text{page}}$ is

$$\begin{aligned} \overline{n}_{\text{page}} &= n_1 \Pr(P_1) + (n_1 + n_2) \Pr(P_2) + \cdots \\ &\quad + (n_1 + n_2 + \cdots + n_N) \Pr(P_N) \\ &= \sum_{i=1}^{N} \sum_{j=1}^{i} n_j \Pr(P_i), \end{aligned} \tag{2}$$

and the average delay in reaching an MS $\overline{d}_{\text{page}}$ is

$$\begin{aligned} \overline{d}_{\text{page}} &= d \Pr(P_1) + 2d \Pr(P_2) + \cdots + Nd \Pr(P_N) \\ &= \sum_{i=1}^{N} id \Pr(P_i). \end{aligned} \tag{3}$$

3. MS mobility model

In the current paging strategy, every BS in the LR area broadcasts the paging message. There is no attempt to use the information about the movements of the MS in a specific area to intelligently page parts of the LR area where the MS is most likely located. We would like to concentrate on vehicular MSs because they are highly mobile compared to pedestrian users, thus generating more LU traffic, due to their high speeds, and more paging traffic per incoming call, due to the larger LR area sizes associated with vehicular MSs.

Usually the MS movements are characteristic of the area the MS is located and of the time of day. In this paper we will concentrate on the cellular grid architecture. The cellular grid architecture is exemplified by a Manhattan City streets model, with the BSs located at the intersections of

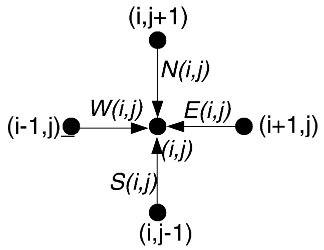

Figure 2. Possible entry directions to cell (i, j).

the streets [8]. In [2] we introduced the *shortest distance* mobility model for vehicular MSs. The model assumes that when traversing an LR area the MS follows the shortest path, measured in number of cells traversed, through the LR area as shown in figure 3. In [2] an iterative algorithm to calculate Pr($v(i,j)$), the probability of visiting cell (i,j), for the *shortest distance* model of mobility, was described. If we define the probability of visiting any cell (i,j) in the LR area from the W, N, E and S directions as

- $W(i,j)$, $N(i,j)$, $E(i,j)$, $S(i,j)$ – the unconditional probability of visiting cell (i,j) from the W, N, E, S directions, as shown in figure 2, using *shortest distance* paths, **within** the LR area, starting at the LU cell,

then Pr($v(i,j)$) can be written as

$$\Pr\bigl(v(i,j)\bigr) = W(i,j) + E(i,j) + N(i,j) + S(i,j).$$

In the *shortest distance* model cell (i,j) can be visited starting at the LU cell only after traversing $|i-\alpha| + |j-\beta|$ cells in the LR area A, where (α, β) denotes the coordinates of the LU cell.

The cell assignments to an LR area shall be represented by a binary $m \times m$ matrix. The matrix X can be written as

$$X = \begin{pmatrix} x_{1m} & x_{2m} & \dots & x_{mm} \\ \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & x_{\alpha\beta} & \dots \\ \vdots & \vdots & \ddots & \vdots \\ x_{11} & x_{21} & \dots & x_{m1} \end{pmatrix}$$

and the elements of the matrix are given by

$$x_{ij} = \begin{cases} 1 & \text{if cell } (i,j) \text{ is} \\ 0 & \end{cases} \begin{Bmatrix} \text{assigned} \\ \text{not assigned} \end{Bmatrix} \quad \text{to the LR area.} \tag{4}$$

The matrix X defines the shape of the LR area. Given matrix X, the probability of visiting each cell in the LR area [2], and the incoming call patterns of the MS, we proceed to calculate Pr(MS in (l,m)), the probability that the MS is located in cell (l,m) at the arrival of an incoming call.

4. System model

Suppose the MS enters the LU cell at time t_0 (figure 3). The MS then traverses M cells in the LR area before exiting

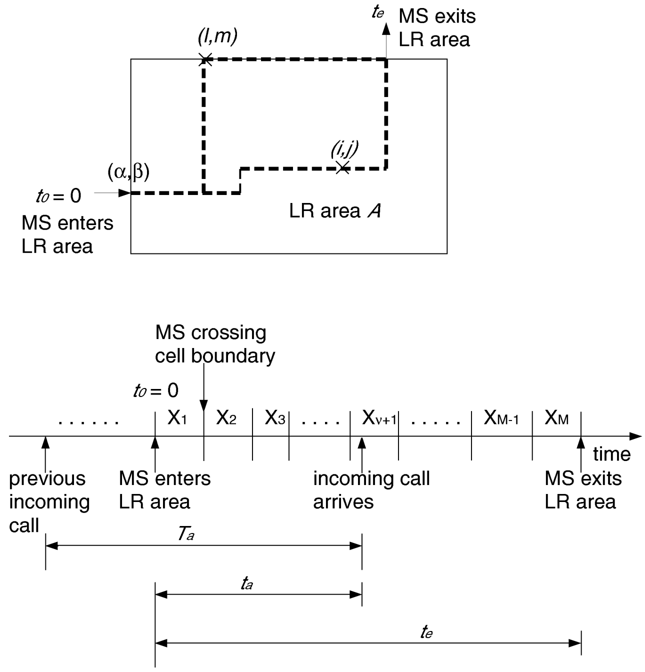

Figure 3. Example of MS movement inside LR area.

at time t_e. The number of cells traversed in the LR will depend on the mobility pattern of the MS inside the LR area, the LR area size and shape. Let us take time t_0 as reference and set $t_0 = 0$. The time the MS spends in each cell depends on the traffic characteristics inside the cell and the cell size. Let the random variable X_i denote the time an MS resides in a cell. Consider a Manhattan City model with the BSs located at the intersection of the streets. At each intersection, there is a traffic light. Let random variable T_i be the time the MS spends moving in a cell and T_d the time the MS is delayed at the intersection, then

$$X_i = T_i + T_d. \tag{5}$$

The times the MS spends in each cell are assumed to be independent identically distributed (i.i.d) random variables. Let t_a denote the time of arrival of an incoming call to the MS, after the MS enters the LR area. Thus we can represent all the time events that occur in the LR area as shown in figure 3.

From figure 3, the time the MS exits the LR area can be written as

$$t_e = \sum_{j=1}^{M} X_j, \tag{6}$$

where M is a random variable representing the number of cells traversed inside the LR area.

Assume that the inter-arrival time between incoming calls, T_a (figure 3), is exponentially distributed with mean $\overline{T}_a = 1/\mu_a$. Thus the probability density function (pdf) of T_a is given by

$$f_{T_a}(t) = \mu_a \, e^{-\mu_a t}.$$

Since we are given that the arrival time of the incoming call occurs after the MS enters LR area A, i.e., after time t_0, referring to figure 3 we have

$$\Pr(T_{\mathrm{a}} > t_0 + t \mid T_{\mathrm{a}} > t_0) = \Pr(T_{\mathrm{a}} > t) \tag{7}$$

due to the memoryless property of the exponential distribution.

Thus the time of arrival of an incoming call after the MS entered the LR area, t_{a}, as shown in figure 3, has exponential distribution with the same mean as T_{a},

$$f_{t_{\mathrm{a}}}(t) = \mu_{\mathrm{a}}\,\mathrm{e}^{-\mu_{\mathrm{a}} t}. \tag{8}$$

5. Pr(MS in (l, m))

To calculate $\Pr(P_i)$, the probability that the MS is located in the ith partition at the time of arrival of the call, we first have to calculate Pr(MS in (l, m)), the probability that the MS is located in cell (l, m), at the time of arrival of the call. In this section we will derive a formula for Pr(MS in (l, m)) that exploits the mobility characteristics of the MS inside the LR area and the incoming call statistics.

For the MS to be located in an arbitrary cell (i, j) in the LR at the time of arrival of the incoming call, at time t_{a}, two conditions have to be satisfied:

1. Cell (i, j) has to lie on the path followed by the MS from LU cell (α, β) to the LR area boundary, and
2. if we denote by ν the number of cells in the LR area traversed by the MS from (α, β) to (i, j), then

   $$\nu = |i - \alpha| + |j - \beta|.$$

   For a call to arrive to the MS while traversing cell (i, j) we must have (figure 3)

   $$\sum_{i=1}^{\nu} X_i < t_{\mathrm{a}} \leqslant \sum_{i=1}^{\nu+1} X_i.$$

Since we are trying to evaluate the performance of the paging algorithm when a call arrives to the MS, we assume that a call will arrive while the MS is in the LR area. Since the above two conditions are independent, the probability that the MS is in cell (i, j) when the call arrives can be written as

$$\begin{aligned}\Pr\big(\text{MS in } (i,j)\big) &= \Pr\big(\text{MS in cell } (i,j) \text{ at } t_{\mathrm{a}} \mid t_{\mathrm{a}} < t_{\mathrm{e}}\big)\\ &= \Pr\big(\text{MS enters cell } (i,j) \textbf{ from } \text{LR area}\big)\\ &\quad \times \Pr\left(\sum_{i=1}^{\nu} X_i < t_{\mathrm{a}} \mid t_{\mathrm{a}} < t_{\mathrm{e}}\right)\\ &\quad - \Pr\big(\text{MS exits cell } (i,j) \textbf{ to } \text{LR area}\big)\\ &\quad \times \Pr\left(\sum_{i=1}^{\nu+1} X_i < t_{\mathrm{a}} \mid t_{\mathrm{a}} < t_{\mathrm{e}}\right).\end{aligned} \tag{9}$$

The first term in equation (9) is the probability that the MS visits cell (i, j) **from** inside the LR area, i.e., using paths in the LR area, multiplied by the probability that the incoming call arrives to the MS after the MS enters (i, j) given that the call arrives to the MS before it exits the LR area. We subtract from the first term the probability that the call arrives to the MS, while still **in** the LR area, but after leaving cell (i, j), which can be written as the probability that the MS leaves (i, j) to a cell **in** the LR area, multiplied by the probability that the call arrives after the MS exits (i, j), given that the call occurs before the MS exits the LR area.

Using the notation defined in section 3, the probability of visiting cells in the LR area starting at the LU cell, can be written as

$$\begin{aligned}&\Pr\big(\text{MS enters cell } (i,j) \textbf{ from } \text{LR area}\big)\\ &\quad = x_{ij}\big(E(i,j) + W(i,j) + S(i,j) + N(i,j)\big),\end{aligned} \tag{10}$$

and

$$\begin{aligned}&\Pr\big(\text{MS exits cell } (i,j) \textbf{ to } \text{LR area}\big)\\ &\quad = x_{i-1,j}E(i-1,j) + x_{i+1,j}W(i+1,j)\\ &\qquad + x_{i,j+1}S(i,j+1) + x_{i,j-1}N(i,j-1).\end{aligned} \tag{11}$$

Using equation (6), if M is given, then for any ν,

$$\begin{aligned}&\Pr\left(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i \mid t_{\mathrm{a}} < t_{\mathrm{e}},\ M\right)\\ &\quad = \Pr\left(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i \mid t_{\mathrm{a}} < \sum_{j=1}^{M} X_j\right).\end{aligned} \tag{12}$$

Let $M(\iota, \kappa)$ represent the number of cells traversed in the LR area before exiting, such that cell (ι, κ) is the last cell visited in the LR area. Denote by $\Pr(\mathrm{e}(\iota, \kappa))$ the probability that cell (ι, κ) is the last cell the MS visits before exiting the LR area [2]. Then $\Pr(M = M(\iota, \kappa)) = \Pr(\mathrm{e}(\iota, \kappa))$. Using the Law of total probability and Bayes theorem,

$$\begin{aligned}&\Pr\left(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i \mid t_{\mathrm{a}} < t_{\mathrm{e}}\right)\\ &\quad = \sum_{\forall(\iota,\kappa)} \Pr\left(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i \mid t_{\mathrm{a}} < \sum_{j=1}^{M(\iota,\kappa)} X_j\right)\\ &\qquad \times \Pr\big(M = M(\iota,\kappa)\big)\\ &\quad = \sum_{\forall(\iota,\kappa)} \Pr\left(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i \mid t_{\mathrm{a}} < \sum_{j=1}^{M(\iota,\kappa)} X_j\right) \Pr\big(\mathrm{e}(\iota,\kappa)\big)\\ &\quad = \sum_{\forall(\iota,\kappa)} \frac{\Pr(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i \text{ and } t_{\mathrm{a}} < \sum_{j=1}^{M(\iota,\kappa)} X_j)}{\Pr(t_{\mathrm{a}} < \sum_{j=1}^{M(\iota,\kappa)} X_j)}\\ &\qquad \times \Pr\big(\mathrm{e}(\iota,\kappa)\big),\end{aligned} \tag{13}$$

where $M(\iota, \kappa) = |\iota - \alpha| + |\kappa - \beta| + 1$.

$\Pr(\mathrm{e}(\iota, \kappa))$ is given by [2]

$$\begin{aligned}\Pr\big(\mathrm{e}(\iota,\kappa)\big) &= x_{\iota\kappa}\big(S(\iota,\kappa+1)\overline{x}_{\iota,\kappa+1} + W(\iota+1,\kappa)\overline{x}_{\iota+1,\kappa}\\ &\quad + N(\iota,\kappa-1)\overline{x}_{\iota,\kappa-1} + E(\iota-1,\kappa)\overline{x}_{\iota-1,\kappa}\big),\end{aligned} \tag{14}$$

where

$$\overline{x}_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 1, \\ 1, & \text{if } x_{ij} = 0. \end{cases}$$

If $\nu \geqslant M(\iota, \kappa)$, then

$$\frac{\Pr(t_\mathrm{a} < \sum_{i=1}^{\nu} X_i \text{ and } t_\mathrm{a} < \sum_{j=1}^{M(\iota,\kappa)} X_j)}{\Pr(t_\mathrm{a} < \sum_{j=1}^{M(\iota,\kappa)} X_j)} = \frac{\Pr(t_\mathrm{a} < \sum_{j=1}^{M(\iota,\kappa)} X_j)}{\Pr(t_\mathrm{a} < \sum_{j=1}^{M(\iota,\kappa)} X_j)} = 1. \tag{15}$$

If $\nu < M(\iota, \kappa)$, then

$$\frac{\Pr(t_\mathrm{a} < \sum_{i=1}^{\nu} X_i \text{ and } t_\mathrm{a} < \sum_{j=1}^{M(\iota,\kappa)} X_j)}{\Pr(t_\mathrm{a} < \sum_{j=1}^{M(\iota,\kappa)} X_j)} = \frac{\Pr(t_\mathrm{a} < \sum_{i=1}^{\nu} X_i)}{\Pr(t_\mathrm{a} < \sum_{j=1}^{M(\iota,\kappa)} X_j)}. \tag{16}$$

Inserting equations (10), (11) and (13)–(16) into equation (9) we obtain a solution for Pr(MS in (l, m)).

In the next subsection we show how to calculate $\Pr(t_\mathrm{a} < \sum_{i=1}^{\nu} X_i)$ for any value ν for a Manhattan City model with the BSs located at the intersections of the streets. Using this result we can completely solve for Pr(MS in (l, m)) at the time of arrival of the incoming call.

5.1. Calculation of $\Pr(t_\mathrm{a} < \sum_{i=1}^{\nu} X_i)$

From equation (8) we found that the time of arrival of an incoming call to the MS after entering the LR area is exponentially distributed with mean $1/\mu_\mathrm{a}$. Thus,

$$f_{t_\mathrm{a}}(t) = \mu_\mathrm{a}\, \mathrm{e}^{-\mu_\mathrm{a} t}.$$

The time the MS spends in a cell is given by equation (5). We will use a Manhattan City streets model, corresponding to the grid architecture, with the length of the streets $L =$ 300 m and the BSs located at the intersection of the streets. Traffic lights control the intersections with the duration of the green cycle $g = 30$ s and red cycle $r = 30$ s [1,11]. The traffic lights are assumed not coordinated. This maintains the assumption of i.i.d X_i. The speed of vehicular MSs in inner city follows a (truncated) Gaussian distribution with mean $\overline{v} = 10.86$ m/s and a standard deviation of $\sigma_v =$ 2.19 m/s [9].

An MS arriving at an intersection during the green cycle of the traffic light will suffer no delay. The proportion of users delayed, P_d, is equal to the ratio of MSs arriving during the red cycle to the total number of MS arriving at the intersection. Assuming uniform flow then,

$$P_\mathrm{d} = \frac{r}{r+g} = \frac{1}{2}$$

and when delayed an MS may be delayed anywhere from 0 to ($r = 30$) s. Thus the pdf of the delay at the intersection can be modeled as

$$f_{T_\mathrm{d}}(t) = \frac{g}{r+g}\delta(t) + \frac{1}{r+g}\big[u(t) - u(t-r)\big], \tag{17}$$

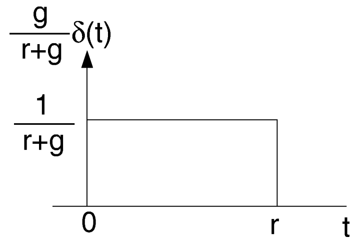

Figure 4. The pdf of the waiting time at an intersection.

where

$$u(t) = \begin{cases} 1, & t \geqslant 0, \\ 0, & \text{otherwise,} \end{cases}$$

and $\delta(t)$ is the dirac delta function.

The speed of the MS moving in the cell has a truncated Gaussian distribution, which can be approximated by a Gaussian distribution with the same parameters. Thus,

$$f_v(t) = \frac{1}{\sigma_v\sqrt{2\pi}}\, \mathrm{e}^{-(t-\overline{v})^2/(2\sigma_v^2)}. \tag{18}$$

The time spent moving in the cell is given by ($v > 0$)

$$T_i = \frac{L}{v}; \tag{19}$$

the mean and variance of T_i can be approximated by [7]

$$\overline{T}_i \simeq \frac{L}{\overline{v}} + \frac{2L}{\overline{v}^3}\frac{\sigma_v^2}{2},$$
$$\sigma_{T_i}^2 \simeq \frac{L^2}{\overline{v}^2} + \frac{6L^2}{\overline{v}^4}\frac{\sigma_v^2}{2} - \overline{T}_i^2.$$

Since T_i and T_d are independent, the mean and variance of X_i is given by

$$\overline{X}_i = \overline{T}_i + \overline{T}_\mathrm{d},$$
$$\sigma_{X_i}^2 = \sigma_{T_i}^2 + \sigma_{T_\mathrm{d}}^2.$$

Since we have assumed that the random variables (RVs) X_i are i.i.d, let

$$\widetilde{X}_\nu = \sum_{i=1}^{\nu} X_i.$$

Therefore,

$$f_{\widetilde{X}_\nu}(t) = f_{X_1}(t) * \cdots * f_{X_\nu}(t),$$
$$\overline{\widetilde{X}}_\nu = \nu\overline{X}_i,$$
$$\sigma_{\widetilde{X}_\nu}^2 = \nu\sigma_{X_i}^2.$$

The central limit theorem states that if the RVs X_i are independent, then under general conditions the density $f_{\widetilde{X}_\nu}(t)$ tends to a normal curve, $N(\nu\overline{X}_i, \nu\sigma_{X_i}^2)$, as $\nu \to \infty$. However, for our case $f_{\widetilde{X}_\nu}(t)$ can be approximated as normal even for small ν (say, $\nu \geqslant 4$) since the pdf of X_i, $f_{X_i}(t)$, is smooth.

For very small values of ν, say $\nu < 4$, the Gaussian approximation of $\widetilde{X}_\nu$ will not be very accurate. However, in this case, t_a will be much bigger than $\widetilde{X}_\nu$, and $\Pr(t_\mathrm{a} < \widetilde{X}_\nu)$

will be approximately zero, irrespective of what the exact pdf of $\widetilde{X}_\nu$ looks like. So for the purpose of calculating $\Pr(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i)$ the pdf of $\widetilde{X}_\nu$ may be considered Gaussian $\forall\nu$. Since t_{a} has an exponential pdf with mean $1/\mu_{\mathrm{a}}$ and $\widetilde{X}_\nu$ is $N(\nu\overline{X}_i, \nu\sigma^2_{X_i})$, therefore

$$\Pr\left(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i\right) = 1 - \Pr\left(\sum_{i=1}^{\nu} X_i \leqslant t_{\mathrm{a}}\right) = 1 - \int_0^\infty \Phi_{\widetilde{X}_\nu}(t)\mu_{\mathrm{a}}\,\mathrm{e}^{-\mu_{\mathrm{a}} t}\,\mathrm{d}t. \quad (20)$$

Note that $\Pr(t_{\mathrm{a}} < \sum_{i=1}^{\nu} X_i)$ is independent of the time of arrival of the incoming call and the time that the MS enters the LR area. The integral in equation (20) can be evaluated numerically for different values of ν and used in equations (15) and (16).

6. Problem formulation

Given the LR area shape and size, the MS mobility model and the MS incoming call statistics we were can calculate Pr(MS in (l,m)) $\forall(l,m)$ when the incoming call arrives, independent of the time of arrival of the call and the time the MS enters the LR area. Using the values of Pr(MS in (l,m)) computed in section 5, the cells (l,m) are mapped into the probability line as shown in figure 1. At the time of arrival of a call, the probability the MS is in partition P_i is

$$\Pr(P_i) = \sum_{\forall(l,m)\in P_i} \Pr\big(\text{MS in } (l,m)\big). \quad (21)$$

The objective of the *selective paging* algorithm is to choose the optimum N and n_i to minimize the number of paging messages subject to a delay constraint.

Problem I.

$$\text{Minimize} \quad \overline{n}_{\text{page}} = \sum_{i=1}^{N}\sum_{j=1}^{i} n_j \Pr(P_i)$$

$$\text{subject to} \quad \overline{d}_{\text{page}} = \sum_{i=1}^{N} id \Pr(P_i) < D_1$$

$$\text{and} \quad \sum_{j=1}^{N} n_j = k.$$

Proposition 1. Problem I is NP-complete.

Proof. We prove that Problem I is NP-complete by restriction. Let $\sum_{j=1}^{i} n_j \Pr(P_i) = s(P_i)$ be the cost associated with choosing partition P_i with size n_i as the ith partition, $i = 1, \ldots, N$. Let $v(P_i) = id\Pr(P_i)$ be the delay cost associated with choosing a particular ith partition P_i. Let U be the set of all partitions and $U' \subset U$ be the subsets containing distinct, mutually exclusive and collectively exhaustive, partitions. Problem I reduces to

$$\text{Find optimal } U' \text{ to minimize} \quad \sum_{P_i\in U'} s(P_i)$$

$$\text{subject to} \quad \sum_{P_i\in U'} v(P_i) \leqslant D_1.$$

This is the Knapsack problem which is NP-complete [4]. One way to solve it is by complete enumeration, varying the number of partitions and enumerating all the possible $\overline{n}_{\text{page}}$ and corresponding $\overline{d}_{\text{page}}$ and finding the optimum number of partitions and partition sizes that minimize $\overline{n}_{\text{page}}$ subject to the paging constraint. □

The number of ways that we can partition k cells into N partitions such that $n_j \geqslant 1\ \forall j$ and $n_1 + \cdots + n_N = k$ is given by [5]

$$\binom{k-1}{N-1}.$$

As k becomes large, the computational complexity of enumerating all the possible partition sizes for a particular N increases. For small N complete enumeration is possible. We expect N to be very small. In fact, in the example we have selected, the optimal N, that achieves significant bandwidth reduction for an acceptable delay, is less than four. In the next example we will investigate how the minimum value of $\overline{n}_{\text{page}}$ and the corresponding delay $\overline{d}_{\text{page}}$ vary as the number of partitions increases and what value of N will be satisfactory for a practical problem.

7. *Selective paging* example

In the following example we will investigate how the minimum value of $\overline{n}_{\text{page}}$, equation (2), for a particular N, and the corresponding value $\overline{d}_{\text{page}}$, equation (3), vary with N. In this example, we assume that the MS moves continuously. Using a similar technique to that described in [2], we can also analyze the situation in which the MS has arrived at its destination and stops for a relatively long time. The *selective paging* algorithm will be investigated for different MS mobility models and incoming call statistics, and different LR area sizes. The following parameters will be used for our example:

- $\widetilde{X}_\nu \sim N(\nu\,\overline{X}_i, \nu\,\sigma^2_{X_i})$.
- Two *shortest distance* mobility models will be investigated, one with $P_{\mathrm{s}} = 0.6$ and $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.2$, the other with $P_{\mathrm{s}} = 0.8$ and $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.1$.
- Two different LR areas, typical for vehicular MSs, will be investigated. The LR area shapes are the optimal rectangular LR area shapes that minimize the LU rate subject to a constraint on the size of the LR area [2]. The area sizes and dimensions of the LR areas for the two mobility models are shown in tables 1 and 2.

Table 1
Optimum rectangular LR area dimensions ($P_s = 0.8,\ P_r = P_l = 0.1$).

Size	Width	Length
30	3	10
120	15	8

Table 2
Optimum rectangular LR area dimensions ($P_s = 0.6,\ P_r = P_l = 0.2$).

Size	Width	Length
33	11	3
120	15	8

For a given k, we will partition the LR area into N partitions, as shown in figure 1. By complete enumeration we find the number of cells in each partition that will minimize $\overline{n}_{\text{page}}$ (equation 2), and the corresponding $\overline{d}_{\text{page}}$ (equation 3).

The first step is to calculate Pr(MS in (l, m)) in equation (9) and to order the cells on the probability line as shown in figure 1. Tables 3–8 show some examples of the ranking of the values Pr(MS in (l, m)) for different cells. For the LR areas shown the LU cell is the cell ranked (1) and the MS enters the LU cell from the W direction. The cells of the tables represent the BS location in the LR area relative to the LU cell, for a grid architecture. The entries in the table are the rank of the probability line of figure 1. For example, the LU cell has the highest Pr(MS in (l, m)), thus it has rank $= 1$ and in table 5 cells at the top and bottom right have the lowest Pr(MS in (l, m)) and thus have rank 120 and 119, respectively. The coordinates of these cells are $(\alpha + 7, \beta + 7)$ and $(\alpha + 7, \beta - 7)$ respectively, where (α, β) is the coordinate of the LU cell. Two interesting observations can be made from the tables:

- Tables 5 and 6 show that even though the dimensions of the optimum LR areas for the two mobility patterns are the same, for the same size constraint, the ranking of the values Pr(MS in (l, m)) on the probability line are very different even for the same incoming call rate. This shows that it is important to consider the MS mobility pattern in the optimization of any paging algorithm.
- Comparing tables 5 and 7, and tables 6 and 8 we note that even though the incoming call rate has doubled, the values (not shown) and the ranking of Pr(MS in (l, m)) for the same mobility models are very similar. The reason is that for our example the time between incoming calls is large compared to the time spent in the LR area, and since an exponential distribution truncated over much less than a time constant is approximately uniform, the results for 1 and 2 calls/hr is about the same. For larger cell sizes, where the MS spends more time traversing the cell, the rank will be different for different call rates.

Figures 5–8 show the minimum values of $\overline{n}_{\text{page}}$, obtained by complete enumeration, and the corresponding $\overline{d}_{\text{page}}$ as

Table 3
Cell ranking for model with $P_s = 0.6,\ P_r = P_l = 0.2$, LR size 33 and $\mu_a = 1$ call/hr.

25	30	33
23	26	29
17	20	19
11	14	13
5	6	9
1	3	2
4	7	8
10	15	12
16	21	18
22	27	28
24	31	32

the number of partitions increases, for a calling rate of 1 call/hr and optimal rectangular LR areas of sizes 33 and 120 cells respectively, for a *shortest distance* mobility model with $P_s = 0.6$ and $P_r = P_l = 0.2$, and area sizes 30 and 120 cells for a *shortest distance* mobility model with $P_s = 0.8$ and $P_r = P_l = 0.1$. Comparing the *selective paging* strategy with the conventional paging strategy, which pages every cell in the LR area regardless of the mobility and call patterns, we can draw several conclusions:

1. Partitioning the LR area results in a significant reduction in the average number of cells paged per incoming call compared to a static broadcast paging scheme. For example, in figure 5, by partitioning the LR area into three partitions the average number of cells paged decreases from 30, in the conventional paging scheme, to 8.84 cells, for $P_s = 0.8$ and $P_r = P_l = 0.1$, i.e., the cost of paging per incoming call has significantly decreased by a factor of 3.4.
2. The smaller the LR area, the larger is the decrease in paging cost using *selective paging* compared to the conventional broadcast scheme for a given mobility and call pattern (compare figures 5 and 7). Current cellular networks define the paging area by the switch boundary, which may include 30–70 cell sites to minimize the load on the switch, so the *selective paging* scheme is expected to have a significant gain.
3. The delay associated with locating an MS in the LR area increases approximately linearly as the number of partitions increases. However, this increase is not as significant as the decrease in paging cost. For example, in figure 6, for 30 cell sites and $P_s = 0.8$ and $P_r = P_l = 0.1$, partitioning the LR area into three partitions increases the average delay in locating the MS from d units, in the conventional broadcast paging scheme, to $1.57d$ units. Thus on the average the user has to wait an extra $0.57d$ units of time for the system to locate the MS. However, this additional delay is not as critical as

Table 4

Cell ranking for model with $P_s = 0.8$, $P_r = P_l = 0.1$, LR size 30 and $\mu_a = 1$ call/hr.

5	8	11	14	17	20	23	26	27	30
1	2	3	6	9	12	15	18	21	22
4	7	10	13	16	19	24	25	28	29

Table 5

Cell ranking for $P_s = 0.6$, $P_r = P_l = 0.2$, LR size 120, $\mu_a = 1$ call/hr.

19	22	37	64	91	110	115	120
96	79	76	87	102	106	113	118
74	63	56	67	81	100	107	112
52	41	38	44	59	82	103	95
24	20	29	34	49	68	92	73
9	12	17	30	43	60	84	51
5	8	13	26	46	70	89	33
1	2	3	4	15	57	98	54
6	7	14	27	47	71	88	32
10	11	16	31	42	61	85	50
25	21	28	35	48	69	93	72
53	40	39	45	58	83	104	94
75	62	55	66	80	99	108	111
97	78	77	86	101	105	114	117
18	23	36	65	90	109	116	119

Table 7

Cell ranking for $P_s = 0.6$, $P_r = P_l = 0.2$, LR size 120, $\mu_a = 2$ calls/hr.

23	26	41	66	93	110	115	120
94	79	76	89	102	106	113	118
74	59	56	64	81	100	107	112
48	37	39	46	61	82	103	98
19	20	29	32	51	70	90	73
9	12	17	30	43	62	84	53
5	8	13	24	44	69	87	35
1	2	3	4	15	55	96	54
6	7	14	25	45	68	86	34
10	11	16	31	42	63	85	52
18	21	28	33	50	71	91	72
49	36	38	47	60	83	104	97
75	58	57	65	80	99	108	111
95	78	77	88	101	105	114	117
22	27	40	67	92	109	116	119

Table 6

Cell ranking for $P_s = 0.8$, $P_r = P_l = 0.1$, LR size 120, $\mu_a = 1$ call/hr.

9	16	53	74	95	110	115	120
66	69	84	93	104	106	113	118
50	55	64	79	89	100	107	112
34	41	47	60	75	88	101	98
24	29	37	44	57	78	91	86
18	21	25	36	45	62	81	72
12	14	19	28	39	52	67	32
1	2	3	5	6	7	8	4
11	13	20	27	40	51	68	31
17	22	26	35	46	61	82	71
23	30	38	43	58	77	92	85
33	42	48	59	76	87	102	97
49	56	63	80	90	99	108	111
65	70	83	94	103	105	114	117
10	15	54	73	96	109	116	119

Table 8

Cell ranking for $P_s = 0.8$, $P_r = P_l = 0.1$, LR size 120, $\mu_a = 2$ calls/hr.

9	18	55	74	95	110	115	120
66	69	84	93	104	106	113	118
48	53	64	79	89	100	107	112
34	41	49	60	75	88	101	98
24	29	37	44	57	78	91	86
16	21	25	36	45	62	81	72
12	14	19	28	39	52	67	32
1	2	3	5	6	7	8	4
11	13	20	27	40	51	68	31
15	22	26	35	46	61	82	71
23	30	38	43	58	77	92	85
33	42	50	59	76	87	102	97
47	54	63	80	90	99	108	111
65	70	83	94	103	105	114	117
10	17	56	73	96	109	116	119

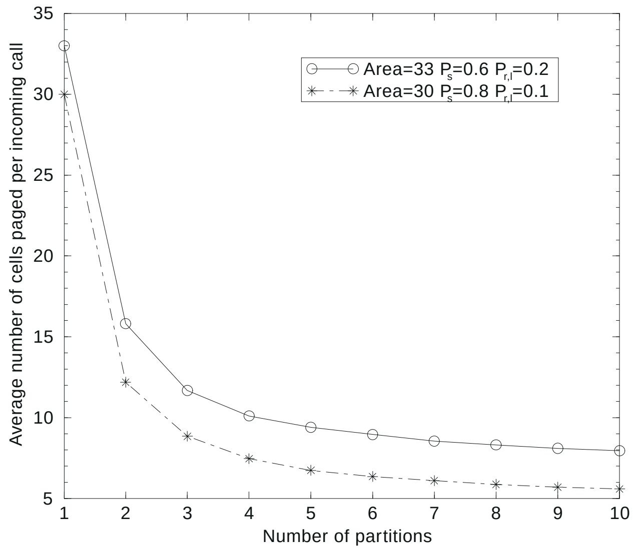

Figure 5. Average number of cells paged for areas with 33 and 30 cells, $P_{\mathrm{s}} = 0.6$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.2$, and $P_{\mathrm{s}} = 0.8$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.1$, respectively.

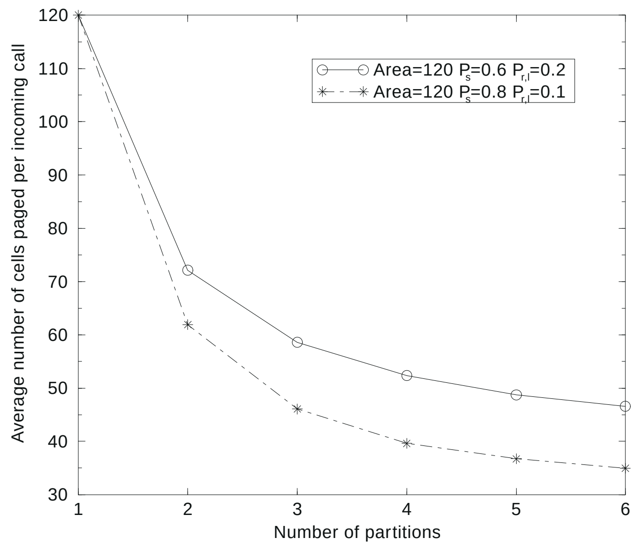

Figure 7. Average number of cells paged for area with 120 cells, with $P_{\mathrm{s}} = 0.6$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.2$, and $P_{\mathrm{s}} = 0.8$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.1$.

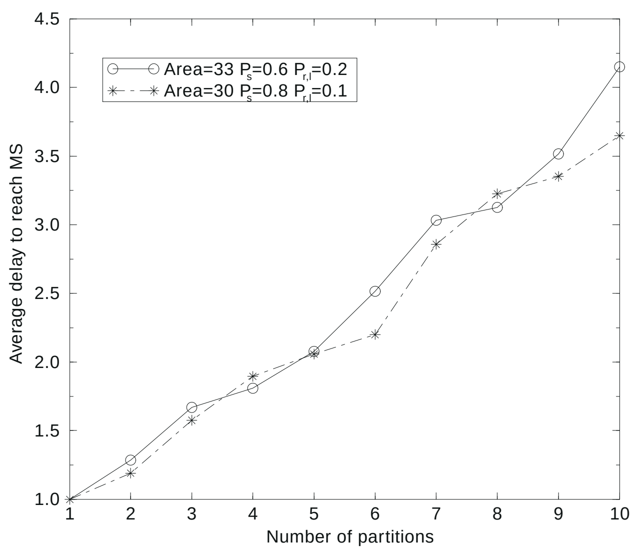

Figure 6. Average delay for areas with 33 and 30 cells, $P_{\mathrm{s}} = 0.6$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.2$, and $P_{\mathrm{s}} = 0.8$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.1$, respectively.

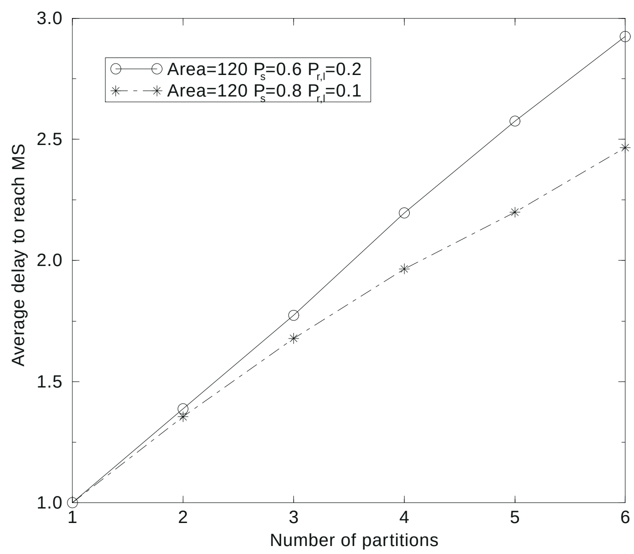

Figure 8. Average delay for area with 120 cells, $P_{\mathrm{s}} = 0.6$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.2$, and $P_{\mathrm{s}} = 0.8$, $P_{\mathrm{r}} = P_{\mathrm{l}} = 0.1$.

the signaling bandwidth reduction achieved, since the bandwidth is the scarce resource. The user may not be very perceptive of the increase in delay, especially if d is small initially.

4. The smaller the LR area, the smaller is the increase in delay for the *selective paging* scheme compared to the conventional broadcast scheme for a given mobility and call pattern (compare figures 6 and 8).

5. Initially as the number of partitions increases there is a significant decrease in the minimum $\overline{n}_{\text{page}}$. But beyond $N = 4$, the decrease in $\overline{n}_{\text{page}}$ is small compared to the initial decrease. On the other hand, there is an approximately linear increase in the corresponding $\overline{d}_{\text{page}}$ as the number of partitions increases. Thus, beyond $N = 4$, the advantages gained from paging cost reduction become less significant compared to the increase in the average delay in searching for an MS in the LR area. So the optimum values of N that achieve significant bandwidth reduction for acceptable delay are not large.

8. Conclusions

In this paper, we proposed a new dynamic *selective paging* strategy that exploits the mobility pattern and incoming call rates of the MS to minimize the paging cost subject to a constraint on the delay in locating the MS. The *selective paging* strategy partitions the LR area into several partitions, independent of the time of entry of the MS into the LR area and the time of arrival of the incoming call. The partitions are paged sequentially until the MS is found.

The *selective paging* strategy uses mobility pattern and incoming call rates of the MS to select the sizes, number of partitions and the cells belonging to each partition to minimize the average number of cells paged subject to the delay constraint. We found that the optimum number of partitions for LR areas of different sizes and different mobility models that achieves significant reduction in paging cost for an acceptable delay is small (the value is $N \leqslant 4$ for our example). The significant reduction in cost makes the *selective paging* strategy a very attractive strategy for reducing signaling on the control channels.

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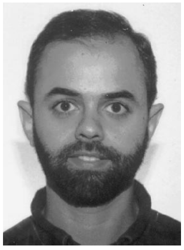

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