GPS-less Localization Protocol for Underwater Acoustic Networks

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Abstract- **The problem of underwater positioning is increasingly crucial due to the emerging importance of sub-sea activities. Knowledge of node location is essential for many applications for which sensor networks can be used. At the surface, positioning problems have been resolved by the extended use of GPS, which is straightforward and effective. Unfortunately, using GPS in the sub-sea environment is impossible and positioning requires the use of special systems. One of the major challenges in the underwater acoustic networks (UANs) area of research is the development of a networking protocol that can cope with the management of a dynamic sub-sea network. We propose a scheme to perform node discovery, using only one seed node (primary seed) in a known position. The discovery protocol can be divided into two parts: First, building up the relative co-ordinate system. Second, involving more remote nodes becoming seed nodes for further discoveries. Four different algorithms have been investigated; (i) Farthest/Farthest Algorithm, (ii) Farthest/Nearest Algorithm, (iii) Nearest/Farthest Algorithm and (iv) Nearest/Nearest Algorithm. We investigated the performances of random network topologies. Different locations of primary seed node were exercised and statistics for node discovery will be reported.**

Fig. 1. Underwater acoustic network

I. INTRODUCTION

Underwater acoustic networks can be formed by acoustically connected anchored nodes, autonomous underwater vehicles (AUVs), and it is possible to have a surface link that serves as a gateway to provide a communication link to an onshore station. Fig. 1 shows a generic underwater acoustic network.

An underwater network has several limitations compared to radio networks, most importantly the propagation delays which are very long with limited bandwidth. Another restriction that needs to be considered in UANs is the incapability of modems to transmit and receive signals at the same time (the near-far effect). To prevent the near-far effect which causes loss of data, scheduled transmission is required. The technique of node discovery must minimize the exchange of data in order to keep network management overheads to a minimum. Furthermore, in underwater acoustic networks, node connectivity is unpredictable. This connectivity depends upon several factors such as relative node orientation, noise level, propagation losses and fading. The connectivity is further affected by relative movement of the nodes, node and link failures and the addition of new nodes. Consequently, a very important characteristic of an underwater communication network is the ability to deal with changing topology.

To achieve full network functionality, nodes need to self-organize in an autonomous network which can adapt to the characteristics of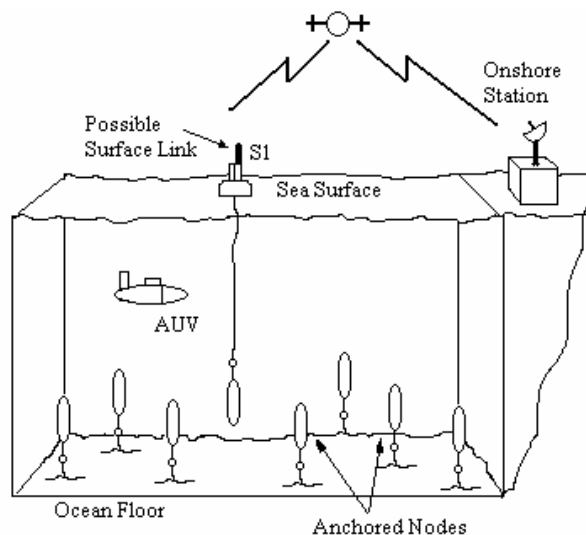 the ocean environment. This paper addresses the following problem: Given a set of nodes with unknown position co-ordinates, determine the relative co-ordinates of nodes.

II. RELATED WORK

In a localization system, several capabilities are necessary. First, the measurement techniques used to gain the information such as distance and other information. Second, the network discovery protocol which concerns the communication between nodes. Finally, techniques of deployment either using the anchor or beacon (nodes with known co-ordinate) or anchor-free bases.

The most popular measurement type is ranging. There are two methods used to obtain range measurements; timing and signal strength. Ranging is usually provided by estimating the distance to a neighbour by measuring the received signal strength (RSS) [1-3] from that neighbour, by time of arrival (ToA) [4] or by time difference of arrival (TDoA) [5].

In the ToA approach, the distance between a remote node and the beacon is measured by finding the one way propagation time between that node and the beacon. Geometrically, this provides a circle, centred on the beacon, on which the remote node must lie. By using at least three beacons to resolve ambiguities, the remote node's position is given by the intersection of the circles. In the TDoA approach, the time

difference of transmission and reception at the beacons is used. By using this approach, the time synchronization can be eliminated [5]. Time of arrival range measurement can be implemented using inquiry-response protocol [6, 7]. Another measurement method for node localization is Angle of Arrival (AoA) [8] where the node estimates the direction from which a neighbour is sending a signal. It can be implemented either using an antenna array, or a combination of radio and ultrasound receivers. In this method, triangulation is used for the localization.

A localization system can be implemented that is based on RSS, ToA, TDoA or AoA, or a combination of these. However, due to a non-uniform signal propagation environment, especially in underwater acoustic networks, RSS methods are not very reliable and accurate. With antenna array is needed in the AoA method; it is impractical to employ in large networks because it is very costly. Furthermore, in this method, nodes may require additional hardware such as a digital compass to provide more information about the node's orientation. Even though ToA or TDoA may require additional hardware at the sensor nodes to receive a signal [9] these methods have better accuracy and are most suitable to be implemented in an underwater environment.

Another requirement for a localization system is the network discovery protocol. There have been many investigations in the radio network field into neighbourhood node and topology discovery [10–12]. In these protocols each node broadcasts a message to gain information of the network. Protocols, such as Bluetooth [13], propose and analyze symmetric protocols for 2-node link formation, which is based on a random schedule. Law et al. [14] and Birthday protocol [11] propose a probabilistic protocol for node discovery; a node decides, with a probability p, to start discovering other nodes, or, with probability 1-p, to listen until it discovered by another node. A node gives up, either if it does not discover another node or does not hear from any other node within a defined period of time. However, these protocols aim at establishing one-to-one connections.

The discovery protocol discussed above may require explicit exchanges of messages containing the node address/ID and, sometimes, the node co-ordinates. Furthermore, the nodes do not share their discoveries with other nodes in the region. This typically requires some form of reliable broadcast system which makes these schemes very expensive in terms of energy consumption and convergence time, matters of high priority in underwater networks.

Previous research has addressed two deployment techniques for localization in ad hoc networks. These are known as anchor-based and anchor-free. Localization algorithms that rely on anchor nodes [15-23] assume that a certain minimum number, or fraction, of the nodes know their position by structured placement or by using some other location mechanism. The advantage of having anchor nodes which are spatially distributed throughout the network region is that they let devices compute their location in a scalable, decentralized manner. For such mechanisms, questions arise as to the number and the sophistication of placements of anchor nodes.

Doherty [15] has proposed a convex optimization technique with the anchor nodes to be placed on the outer boundary, preferably at the corners of the deployment area to work well. The advantage of this approach is that it requires very few anchors (3 or 4) since all system constraints are solved globally. However, this algorithm is not very robust to failures when there are ambiguities in measurements. The Cricket Location Support System [16], Active Badge [17], the Bat System [18] and HiBall Tracker [19] use proximity based techniques and propose guidelines for the deployment of anchor nodes based on practical considerations (influenced by environment conditions and application requirements). The anchor nodes are located in an unobtrusive location like a ceiling or wall. Another approach to addressing the deployment problem of anchor nodes is using optimal placement algorithms including Pursuit-Evasion [20] and Facility Location [21, 22].

In contrast, the anchor-free method [23] uses local distance information to attempt to determine node co-ordinates. In this method every node in the network performs discoveries and shares the information with neighbouring nodes and, thus, defines the local co-ordinate system and finally the network co-ordinate system.

Nevertheless, the techniques discussed in the deployment system above are (a) not scalable to large sensor networks, and (b) not suitable for rapid deployment. In addition, with the limitations in such underwater acoustic networks as mentioned earlier, it is impossible to employ anchor nodes that infer their position through GPS. In our method, we do not use any anchor nodes in the network except the primary seed node (node with known co-ordinate). Information received during discovery is shared with neighbouring nodes and the information is then used to determine second order seed nodes.

III. DISCOVERY PROTOCOL AND LOCALIZATION ALGORITHM

To establish the relative co-ordinate system for the network, the protocol proposed in this paper uses various commands for peer to peer communication. Discovery and localization protocol can be divided into two parts: Stage 1: Building up the relative co-ordinate system using the information gained from the first three seed node discoveries. Stage 2: Further node discovery by selected seed nodes.

Assume that S_1 is the first seed node and there are remote nodes available in its region of communication. Following node deployment, seed node S_1 will broadcast a DISC_COMM packet. It will await replies from nodes within its range. When replies are received, information such as node ID and distance are retained in the seed node memory. In this first discovery, the seed node only discovers the node IDs and their distances but not their location. The next stage is to set a second seed node for further discovery. We propose that the second seed node selected will be the farthest node from S_1. The advantage of choosing the farthest node as the second seed node, S_2, is that a larger area can be covered more quickly. Assume that A_i is the information set of a discovery sequence, it contains the distance measurement and node ID of those nodes replied. S_1 will broadcast A_1 and MORE_DISC to its

neighbouring nodes. At this point, each node in the S_1 region has the information of A_1. If a node in the S_1 region receives this command and the ID is equal to the node ID of the next seed node, then this node will recognise that it is to become the second seed node, S_2. S_2 proceeds with the same manner of discovery; it will then broadcast the newly discovered information, A_2, back to its neighbours. The neighbouring nodes that receive this information will store the new information in their memory. Assuming that there is no data loss during broadcasting, after receiving information from S_2, S_1 will then update its own neighbours by re-broadcasting the A_2 data. At this point each node in the S_1 and S_2 regions has the information of A_1 and A_2. At this juncture, the locations of any overlap nodes from S_1 and S_2 are ambiguous. In order to solve this ambiguity, we introduce a third seed node, S_3. S_3 is chosen from those nodes that lie in both the S_1 and the S_2 regions and have the maximum summation distance from S_1 and S_2. After selecting S_3, S_1 will send another MORE_DISC command to define S_3. S_3 will then start a new discovery process by broadcasting a DISC_COMM command. After it receives replies from neighbourhood nodes, it rebroadcasts the information, A_3, back to its neighbours. Since S_1 and S_2 are in the region of S_3, when they receive the new information from S_3 they immediately broadcast the information to their own neighbours.

Fig. 2 shows the regions of two and three distance measures after discovery by the first three seed node. The grey area in this figure shows the area that has knowledge of three distance measures of S_1, S_2 and S_3. Consider that S_1 has absolute knowledge of its own coordinate defined here as 0, 0. S_2 will be assumed to be at d_{12}, 0 coordinate, where d_{12} is the distance of the farthest replying node from S_1. With S_1 being the origin of the relative coordinate system, S_2 is defined to lie on the positive x axis. S_3 is now assumed to have a positive y component to define the y axis. With the assumptions made and information received, nodes in the overlap region are able to calculate their own coordinates and the coordinates of other nodes using the triangulation technique. Table I shows the summaries this approach made.

The cross-hatched region in fig. 4 shows the area where only two known distance measures from their seed nodes are certain. As there are two solutions from this method, it is essential to know on which side of the line the nodes lie. This is the drawback with only two distance measurements, where we have the ambiguity of node placement. This may be resolved using the method described below. The computation of the coordinates will be done locally at each node.

TABLE I
RELATIVE CO-ORDINATE SYSTEM

Seed Node	x-Coordinate	y-Coordinate
First, S_1	0	0
Second, S_2	d_{12}	0
Third, S_3	$d_{13}\cos\theta$	$d_{13}\sin\theta$

$$\theta = \cos^{-1}\left(\frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2 \times d_{12} \times d_{13}}\right)$$

d_{12} – distance between first and second seed node
d_{13} - distance between first and third seed node
d_{23} – distance between second and third seed node

Assuming that:

$M_d = \text{maximum Distance Broadcast}$

$NR_1 = \text{Node Reply from } S_1; NR_2 = \text{Node Reply from } S_2;$
$NR_3 = \text{Node Reply from } S_3$
and
$NR_{12} = NR_1 \cap NR_2 \in NR_3; NR_{13} = NR_1 \cap NR_3 \in NR_2;$
$NR_{23} = NR_2 \cap NR_3 \in NR_1$

Algorithm for 2 Known Coordinates with S_1
and S_2 as Reference Nodes

If $NR_{12} \sim= \Phi$
   Compute Possible Locations of NR_{12}
   $Z_{1,i} = X_{1,i}; Y_{1,i}$ and
   $Z_{1,j} = X_{1,j}; Y_{1,j}$

   Compute Distance from $Z_{1,i}$ and $Z_{1,j}$ to S_3;
   $d_{1,i}$ and $d_{1,j}$ respectively

   If $d_{1,i} > M_d \,\&\, d_{1,j} < M_d$
     $NR_{12} = Z_{1,i}$
   end
   If $d_{1,i} < M_d \,\&\, d_{1,j} > M_d$
     $NR_{12} = Z_{1,j}$
   end
   If $d_{1,i} > M_d \,\&\, d_{1,j} > M_d$
     %No Possible Coordinate can be Calculated
     $NR_{12} = \Phi;$
   end
   If $d_{1,i} < M_d \,\&\, d_{1,j} < M_d$
     %No Possible Coordinate can be Calculated
     $NR_{12} = \Phi;$
   end
else, end

Similar algorithms can be applied to NR_{13} and NR_{23} to gain the relative location for the nodes in their region.

IV. ALGORITHMS FOR SELECTING FURTHER SEED NODES

A. Farthest/Farthest Algorithm

The Farthest/Farthest algorithm uses the farthest undefined node from a previous seed node, and the node with the maximum summation distance from this node and the previous seed node. Each remote node in the seed nodes region of the first stage of discovery will independently compute the relative location of all other nodes. Because of a lack of sufficient data, some nodes will be unable to fully define their location. Therefore, more information, such as distances from nodes with known co-ordinates, is needed for them to gain their relative co-ordinates.

In this Farthest/Farthest algorithm, first, each node in the seed nodes region will identify the undefined node in their dataset and find the farthest node from their seed node. If a node determines that it is the farthest undefined node from their

seed node, it will automatically set itself as a new seed node and carry out a discovery process. When it receives replies from its neighbouring nodes, it will define its own relative co-ordinates and re-broadcast the information back to its neighbours. At this stage, the positions of overlap nodes between the new seed nodes and their first stage seed nodes are ambiguous. Therefore, another seed node is needed in order to solve the ambiguity. This seed node can be defined as the maximum summation distance of undefined node between the two seed nodes. The process will end when the seed node receives replies from all nodes with coordinates in its region, or the seed node cannot find its own coordinates where only one distance measurement of a node with known coordinates replies during the discovery.
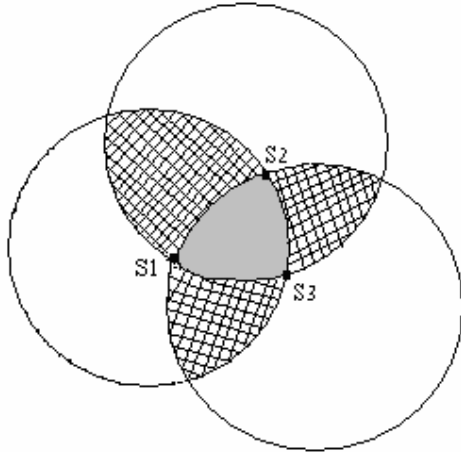

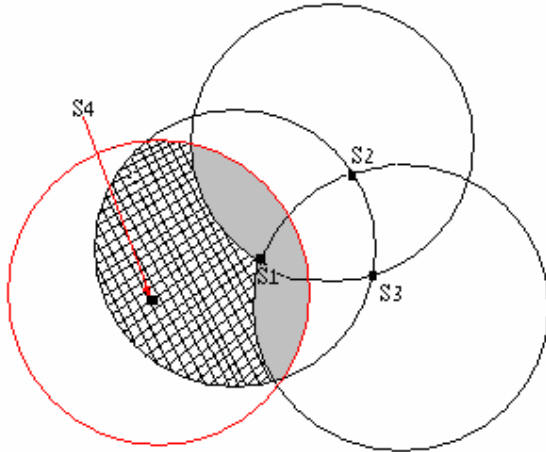Fig. 2. Region of two and three distance measurements

Fig. 3. Area of nodes with known co-ordinate by S_4 discovery and potential area of the S'_4

Consider fig. 4 as an example. S_4 is assumed to be the farthest node from S_1, therefore S_4 becomes the next seed node. S_4 precedes the same procedure of discovery by broadcasting DISC_COMM and waiting for reply from other nodes in its

region. When it receives all the replies from the nodes, it will re-broadcast the discovery information back to its neighbours and use the discovery information to determine its own coordinates. If the seed cannot define its own relative co-ordinate then the next farthest node of the undefined node in the S_1 region is used as the new seed node. The discovery process will carry on until the new seed node resolves its own coordinates. The remote nodes in a region discovered by S_4 may contain one (from S_4) or two known distance measures (from S_1 and S_4, say). With this information, all the nodes in the cross-hatched area shown in figure 3 still do not have sufficient data to solve their location, since there is ambiguity of the nodes' position. Following this problem, another seed node is needed. The next chosen seed node, S'_4, will be the undefined node with maximum summation distance from S_1 and S_4.

B. Farthest/Nearest Algorithm

A different approach can be taken in order to gain the relative coordinates of nodes. The Farthest/Nearest algorithm uses the farthest undefined node from a previous seed node and the node with minimum summation distance from this node and the previous seed node.

C. Nearest/Farthest Algorithm

Alternatively, the Nearest/Farthest algorithm can use the nearest undefined node from a previous seed node and the node with maximum summation distance from this node and the previous seed node.

D. Nearest/Nearest Algorithm

The Nearest/Nearest algorithm uses the nearest undefined node from a previous seed node and the node with minimum summation distance from this node and the previous seed node.

V. SIMULATION SET-UP AND PERFORMANCE RESULTS

In this set of experiments, we generated a set of 30 to 100 nodes randomly in a $10 \times 10\ km$ area. The distances between nodes are set not less than $100\ m$ apart. At the initial stage of discovery nodes have no knowledge of location with respect to the other nodes and number of remote nodes in the network. We generated 100 samples and we used the same network topologies for all four algorithms in selecting the next seed node, as described in section IV.

Fig. 4 shows the average of network set-up times for the four algorithms with different numbers of node deployment. It is clear that the network set-up time achieved by all the algorithms increases linearly with the number of nodes in the network. The figure suggests that, with lower numbers of node deployment, the Nearest/* algorithms (*) uses less time for network set-up compared to the Farthest/* algorithms. As expected the Farthest/* algorithms have least performance with the low numbers of node deployment. However, with high numbers of node deployment, the Farthest/* algorithms have a better performance (about 4%−13% less network set time)

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* indicates both Farthest and Nearest

compared to the Nearest/* algorithms. Fig. 5 shows the average number of seed nodes for four algorithms with different numbers of node deployment. The figure suggests that, with lower numbers of node deployment, the Farthest/* algorithms use more nodes to become seed nodes for further discovery compared to the Nearest/* algorithms. As expected the Farthest/* algorithms have least performance with the low number of node deployment. However, with high numbers of node deployment, it is obvious that the Farthest/Farthest algorithm has a better performance compared to the other algorithms. Fig. 6 shows the average number of undefined nodes for four algorithms with different numbers of node deployment. The figure suggests that the average number of undefined nodes increases with the number of node deployment. As expected the Farthest/* algorithms gained better performance compared to the Nearest/* algorithms.

Also investigated were the performances of the algorithms with different locations of a primary seed node. Fig. 7 shows the average of network set-up times for the four algorithms with different numbers of node deployment with primary seed node located at 1000, 5000. The figure suggests that, with lower numbers of node deployment, the Nearest/* algorithms use less time for network set-up compared to the Farthest/* algorithms. As expected the Farthest/* algorithms have least performance with the low number of node deployment. However, with high numbers of node deployment, the Farthest/Farthest algorithm has a better performance (3%─12% less network set up time) compared to the other algorithms. Fig. 8 shows the average number of seed nodes for four algorithms with different numbers of node deployment and with primary seed node located at 1000, 5000. The figure suggests that the Farthest/Nearest algorithms use more nodes to become seed nodes for further discovery compared to the other algorithms.   Fig. 9 shows the average number of undefined nodes for four algorithms with different numbers of node deployment with primary seed node located at 1000, 5000. The figure suggests that the average number of undefined nodes increases with the number of node deployment. As expected the Farthest/* algorithms gained better performance compared to the Nearest/* algorithms.

Our first experiment compares the four algorithms in different performance matrices and studies the impact of different locations of primary seed node in a random topology. The experiment results suggested that the distribution of nodes in the area affects the performance of the algorithms. For larger numbers of node deployment, the Farthest/Farthest algorithms took less time for the network set-up, used fewer seed nodes for discovery and resulted in fewer numbers of undefined nodes compared to the Nearest/* algorithms. It also shows that the performance results vary with different locations of the primary seed node.

VI. CONCLUSION

We have presented a node discovery protocol and localization for UANs. The discovery protocol and localization algorithms proposed here form one of the possible approaches to collaborative location discovery. What is unique in our protocol is that we do not use any anchor node except the primary seed node and use the information gained during the discovery to select the next seed node. Furthermore, in this proposed protocol it is only the seed node that attempts the discovery and the information received is shared among the neighbourhood. However, the proposed protocol and algorithms show that the nodes only know their relative co-ordinates from the primary seed node. We conclude that the Farthest/Farthest algorithm is suggested as having better performances compared to the other algorithms. In addition, the location of the primary seed node can affect the performances of the algorithms. We suggest that the primary seed node located at the centre of the network achieves better performances.

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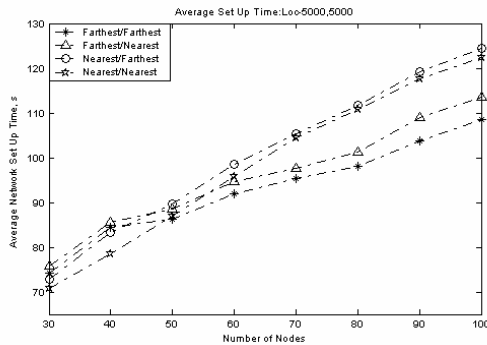
Fig. 7. Average network set up time for random topology with primary seed co-ordinated at 1000, 5000

Fig. 8. Average number of seed nodes for random topology with primary seed co-ordinated at 1000, 5000

Fig. 4. Average network set up time for random topology with primary seed co-ordinated at 5000, 5000
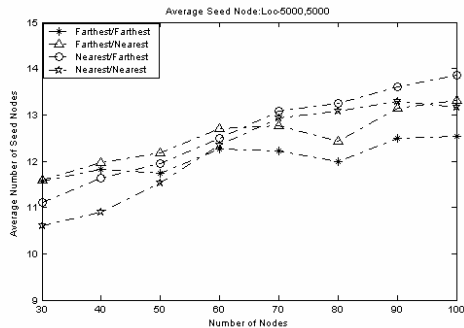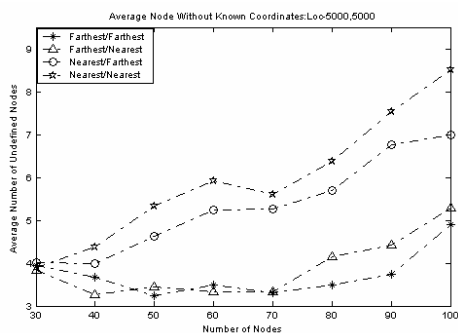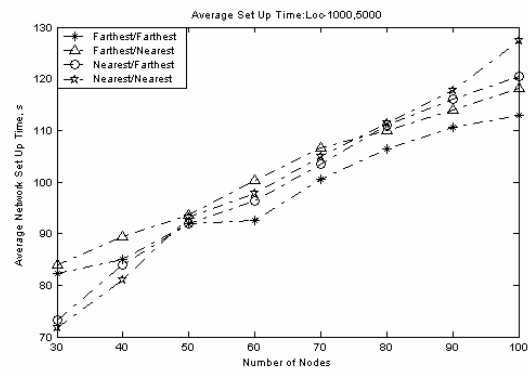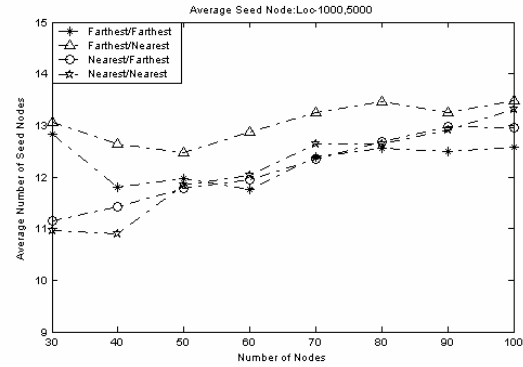

Fig. 5. Average number of seed nodes for random topology with primary seed co-ordinated at 5000, 5000
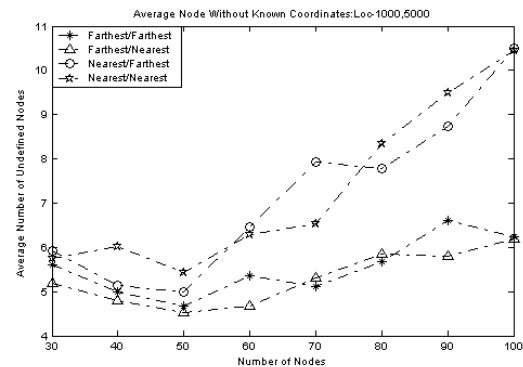

Fig. 9. Average number of undefined nodes for random topology with primary seed co-ordinated at 1000, 5000

Fig. 6. Average number of undefined nodes for random topology with primary seed co-ordinated at 5000, 5000